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A NEW RECONSTRUCTION OF ZENO'S *FLYING ARROW*

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Part I. A survey of important differences in proposed reconstructions of Zeno's *Flying Arrow* argument

The third of Zeno's four arguments against motion, the so-called *Flying Arrow*, is known primarily through Aristotle's *Physics* 239^{b}5-7. Having examined the texts of the argument in nearly all relevant manuscripts of the *Physics* as well as virtually all the extant paraphrases of the argument formulated by Aristotle's ancient commentators, Bekker, in his 1831 edition of Aristotle's works, presented the following text as what he himself took to be the original passage of the *Physics*:[1]

Ζήνων δὲ παραλογίζεται· εἰ γὰρ ἀεί, φησίν, ἠρεμεῖ πᾶν ἢ κινεῖται, ὅταν ᾖ κατὰ τὸ ἴσον, ἔστι δ᾽ἀεὶ τὸ φερόμενον ἐν τῷ νῦν, ἀκίνητον την φερομένην εἶναι ὀϊστόν.

There are five points on which scholars differ in their formulation and analysis of Zeno's *Flying Arrow* argument. Some of these differences affect their interpretations of the argument and some do not. All five points are nevertheless crucial to a correct understanding of the argument, so we will examine each of them carefully. Only when we have analyzed Zeno's argument as a whole will we be in a position to evaluate these historically controversial and important points.

1. *The logical stucture of the Flying Arrow argument and the modal status of its premisses*[2]

Arisototle's formulation of the *Flying Arrow* argument is couched in indirect discourse.[3] Such formulations are indeterminate in ways that sometimes bear importantly on textual interpretation. Consider a simple example: John said that grass is green and snow is white. Does this sentence mean that John made one complex statement (*grass is green and snow is white*) or two simple statements (*grass is green* **and** *snow is white*)? The sentence is itself indeterminate with respect to these two analyses. When necessary, one can eliminate such indeterminacies by explicitly heeding the object-language/metalanguage distinction of modern logic and semantics. (The *object language* is the language about which one is discoursing; the *metalanguage* is the language in which the discourse is being conducted.) In our example, the object language and metalanguage happen to be the same language, viz., English. Similarly, in the *Flying Arrow* argument, Greek is both object language and metalanguage. In such cases, it is particularly useful to mark the object-language/metalanguage distinction explicitly. For example, we could paraphrase our sample sentence by either a. or b. below

a. **John said that** 'grass is green and snow is white'.

b. **John said that** 'grass is green' **and that** 'snow is white'.

according as we interpret John to have made one complex statement or two simple statements. Because the two simple statements are jointly logically equivalent to the one compound statement, it would be a rare context in which the indeterminacy of our sample sentence would matter. But as we will show in detail below, analogous indeterminacies

in Aristotle's formulation of the *Flying Arrow* affect crucially the analysis and interpretation of this famous ancient argument.

Let A, B and C denote the following propositions in Bekker's edition of *Physics* 239^{b}5-7:

A. [εἰ γὰρ] ἀεί [φησίν] ἠρεμεῖ πᾶν ἢ κινεῖται, ὅταν ᾖ κατὰ τὸ ἴσον,

B. ἔστι [δ᾽] ἀεὶ τὸ φερόμενον ἐν τῷ νῦν,

C. ἀκίνητον τὴν φερομένην εἶναι ὀϊστόν.

Below are provisional English translations of the above three Greek propositions. In these translations, boldface type is used for expressions in the metalanguage. Where the object-language/metalanguage status of an expression is indeterminate, we have rendered the expression in both boldface and ordinary type. The phrases κατὰ τὸ ἴσον and ἐν τῷ νῦν are left untranslated for the nonce, but their respective literal transcriptions 'against what is equal' and 'at an instant' may help the reader grasp the meanings of the following partial translations of the above three Greek sentences:

A. **For he (Zeno) says [if** if] everything is always either at rest or in motion when it is κατὰ τὸ ἴσον
B. **[and** and] what moves is always ἐν τῷ νῦν
C. **[then** then] the moving arrow is motionless.[4]

It is evident, at least from the presence of the typically Aristotelian phrases κατὰ τὸ ἴσον and ἐν τῷ νῦν,[5] that in presenting Zeno's argument Aristotle is not *quoting* Zeno's words but rather is *using Greek* to talk about an argument formulated *in Greek* by Zeno. Since this is a far-from-trivial case where object language and metalanguage coincide, it will be important to explicitly mark the object-language/metalanguage distinction when analyzing and interpreting Zeno's argument.

First, (ignoring the phrase 'for he says') we note that there are two ways to understand how the form or structure of Zeno's argument is presented by Aristotle, namely: as a single complex conditional with a conjunctive antecedent (A & B) ⇒ C, or as an inference (a syllogism) from two premisses A and B to the conclusion C. In the former case, the Greek particles εἰ and δέ belong to the object-language, whereas in the latter case they belong to the metalanguage. This observation leads to the following two translations (we continue to leave the phrases κατὰ τὸ ἴσον and ἐν τῷ νῦν untranslated, deferring discussion of their translation to the next section):

(1) **Zeno argues fallaciously, for he says that** '*if everything is always either at rest or in motion when it is* κατὰ τὸ ἴσον *and what moves is always* ἐν τῷ νῦν*, then the flying arrow is motionless*'.

(2) **Zeno argues fallaciously, for he says that if** '*everything is always either at rest or in motion when it is* κατὰ τὸ ἴσον' **and** '*what moves is always* ἐν τῷ νῦν'**, then** '*the flying arrow is motionless*'.

In translation (1), what Aristotle reports Zeno to have said has the logical form of a complex conditional with a conjunction as antecedent:

(A & B) ⇒ C,

whereas in translation (2) it has the logical form of an inference from two premisses:

A, B ⊢ C.[6]

Comparison of the foregoing two versions of Zeno's argument would be facilitated if both were expressed as conditional formulas as in translation (1), or if both were formulated as inferences as in translation (2). For ease of comparison and discussion, we elect to formulate both of them as conditionals. To reformulate the

argument in translation (2) as a conditional, we simply make two applications of what logicians call *Conditional Proof*:

From the two-premisses inference

$$A, B \vdash C,$$

by one application of Conditional Proof we obtain the one-premiss inference

$$A \vdash B \Rightarrow C,$$

whereupon by a second application of Conditional Proof we obtain (as an inference from no premisses) the conditional

$$\vdash A \Rightarrow (B \Rightarrow C)$$

as our reformulation of Zeno's argument in translation (2). Having reformulated the argument in translation (2) as a conditional formula, we then obtain T1 and T2 as the respective object-language formulations of translations (1) and (2):

T1: If everything is always either at rest or in motion when it is κατὰ τὸ ἴσον and what moves is always ἐν τῷ νῦν, then the flying arrow is motionless.

T2: If everything is always either at rest or in motion when it is κατὰ τὸ ἴσον then, if what moves is always ἐν τῷ νῦν, then the flying arrow is motionless.

Most translators adhere closely to the Greek original, so one might uncritically expect them to prefer T1 to T2, i.e., to choose T1 over T2.[7] But by failing to mark object-language/metalanguage distinctions in their translations, these translators effectively import the indeterminacy of δέ from the original Greek text into their translations, e.g., the connective 'and' in their English translations can be understood either as part of an object-language exposition of Zeno's argument or as part of a metalinguistic commentary on an object-language version of the argument. There are,

however, translators who insert the connective 'if' after the connective 'and', thereby decoupling B from A, a move that strongly suggests that they understand the argument (if it were to be expressed as a conditional in the object language) after the fashion of T2.[8]

Three comments are in order. First, from a purely logical point of view, the difference between translations T1 and T2 is inconsequential because their two underlying formulas are logically equivalent, i.e., in classical propositional logic, the conditional formulas (A & B) $\Rightarrow$ C and A $\Rightarrow$ (B $\Rightarrow$ C) are inter-deducible and semantically equivalent. Second, it was Aristotle's practice to present arguments in the form of a syllogism, which according to his technical notion contains two premisses. From a purely historical point of view, therefore, translation T2 seems preferable because it is derived from a syllogism containing two premisses, whereas T1 is derived from an argument having only one premiss. But, third and finally, there is an independent and for us more compelling reason for favoring translation T2 over translation T1, viz., T2 enables interpreters to discuss more easily the modal status of A and B, a consideration that will turn out to be essential for a correct interpretation of the *Flying Arrow*.

It is well known that in conditionals the Greek εἰ, when combined with present indicative mood, is used in the following three ways:

(1) To state a condition, with nothing implied as to its fulfillment,[9]

(2) To cite a fact as a ground of argument or appeal,[10] or

(3) To express a counterfactual as the basis for an inference.[11]

In English the connective 'if" is used in ways (1) and (3), whereas the connective 'since' is used in way (2). So, depending on how one translates εἰ into English, one can explicitly differentiate between uses (1) and (2) of the Greek particle εἰ, i.e., between 'if' and

'since'. We note that the English connective 'if' governs a verb in the indicative mood when used in way (1), whereas it normally governs a verb in the subjunctive mood when used in way (3).

By the *modal status* of a sentence, we refer to the above three ways in which the antecedent of a conditional sentence may be used. So, even if we were dealing with an argument containing just one premiss introduced by εἰ, we would still confront a decision about its modal status, i.e., we would have to decide in which of the above three ways the premiss should be translated. We have chosen to understand the *Flying Arrow* as an argument with two premisses, so relative to premiss modality there are mathematically 3^2 or nine ways in which it can be translated.

We illustrate (some of) this theoretical multiplicity by Cornford's translations of two passages from the *Physics* that are syntactically isomorphic to the *Flying Arrow* argument (when it is expressed as a complex conditional in the object language). Cornford is always careful to make clear by his translation what he takes the modal status of a proposition to be, something the Greek indicative mood leaves indeterminate. Thus, in his translation of 232[a]19 Cornford distinguishes the explicit εἰ from the implicit εἰ (tacitly introducing the second premiss) by using 'if' for the first (explicit) εἰ and 'since' for the second (implicit) εἰ:[12]

> 'But **if** on the other hand we admit that every distance or motion is divisible, so must the corresponding periods of time be, **since** a thing moving at a uniform velocity will cover a part of any distance in less time than the whole.' [*boldface added*]

We pause to emphasize that in Cornford's transformation of the inference underlying 232[a]19 into a corresponding complex conditional, the single εἰ that introduces the premisses is 'doubly' translated: first as 'if' (at the head of the first antecedent) and then as 'since' (at the head of the second antecedent). The correctness of this translation turns on the aforementioned semantics of 'if' and 'since' in English. To see this, imagine a mother trying to show her son how mathematical truths can be useful in ordinary life. She says to the child: '(Suppose) you have two candies in one of your pockets and two more in the other pocket, and two plus two equals four. Then you must have four candies'. Now, translating into English what the mother said to her son as a complex conditional,[13] a linguist would write: '*If* you have two candies in one of your pockets and two more in the other pocket, then, *since* two plus two equals four, you have four candies'. It would be inappropriate and misleading for the linguist to use 'if' in both italicized places; to do so would suggest that the mother had allowed for the possibility that two plus two does not equal four, a possibility that 'since' rules out.

In *Physics* 204[b]14 Cornford takes both premisses to be counterfactuals and so translates the Greek indicative in them by the English subjunctive:

> 'For however relatively feeble the power of one element (say, air) might be with respect to that of another (say, fire), yet, **if fire were limited**, **the air unlimited** in quantity, **and if there were** any ratio of equivalence whatever between the assimilative power of a given volume of air and that of an equal volume of fire, **then** obviously the unlimited volume of air must vanquish and destroy the limited volume of fire.' [*boldface added*]

Not only is Cornford's translation obviously correct, but there is no better example for showing that εἰ + present indicative not only *can be* but sometimes *must be* translated as counterfactual. The determination of the modal status of a proposition depends on its interpretation. In the given place (*Physics* 204^{b}14) a counterfactual translation is the only defensible modal-status determination, because it is clear from context that Aristotle rejects the idea that fire is limited, the air unlimited, in quantity. Another example, which we shall cite below, is Herodotus III 62. There are cases, of course, in which context is not so unambiguously clear as in these two examples. Aristotle's sentence presenting the *Flying Arrow* argument is such a case.

Were there a general practice among translators and interpreters of more fully exploiting the resources of the language they translate into (in the manner of Cornford), one would know from their translations what they take the modal status of the translated propositions to be, e.g., the modal status of the premisses of the *Flying Arrow* argument. As one would expect, Cornford is clear about modal status in the *Flying Arrow*: the fact that he introduces each of its premisses by 'since' shows that he believes that Aristotle thinks of them as propositions that Zeno accepted as true. However, the great majority of translators preserve—wittingly or unwittingly—the indeterminateness of the Greek text, thereby bequeathing the determination of the modal status of the premisses to commentators and interpreters (who are sometimes the translators themselves!). Although we do not ourselves here champion either of these approaches to translation over the other, it is important to recognize that differences in the modal status of the premisses of the *Flying Arrow* will inevitably show up, sooner or later, either in translation or in interpretation.

Our election to formulate Zeno's argument as a conditional rather than as an inference is not unproblematic. It facilitates comparison of translations but it does this at some cost.[14] In particular, it deflects attention from the modal status of Zeno's premisses. When Zeno's argument is expressed as an inference from two premisses

A, B ⊢ C,

the modal status of the premisses A and B can hardly be ignored. Does Zeno intend A (alternatively, B) to state a condition with nothing implied as to its fulfillment, or is he citing a fact as a ground of argument or appeal, or is he making a counterfactual assumption as the basis for an inference? Without these modal determinations, one does not have a well-defined formulation of the *Flying Arrow*.

Consider now T2, the translation of the *Flying Arrow* produced by taking Zeno's argument to have been a two-premisses syllogism. Each premiss of this syllogism can be understood modally in three possible ways, and the same therefore holds for their T2 counterparts, viz., the two antecedents in the conditional T2.[15] So, by distinguishing 'if' from 'since' in the manner indicated above, we obtain the following *nine* materially different variants of T2 (the translation of the *Flying Arrow* in which Zeno's argument, understood as a syllogism or inference from two premisses, is formulated as a conditional with an embedded conditional as consequent):[16]

T2$_1$: **If** everything is always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, if** what moves is always ἐν τῷ νῦν, **then** the flying arrow is motionless.

T2$_2$: **If** everything is always either at rest or in motion when it is κατὰ τὸ ἴσον**, then, since** what moves is always ἐν τῷ νῦν, **then** the flying arrow is motionless.

T2$_3$: **If** everything is always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, if** what moves **were** always ἐν τῷ νῦν, **then** the flying arrow would be motionless.

T2$_4$: **Since** everything is always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, if** what moves is always ἐν τῷ νῦν, **then** the flying arrow is motionless.

T2$_5$: **Since** everything is always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, since** what moves is always ἐν τῷ νῦν, **then** the flying arrow is motionless.

T2$_6$: **Since** everything is always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, if** what moves **were** always ἐν τῷ νῦν, **then** the flying arrow would be motionless.

T2$_7$: **If** everything **were** always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, if** what moves is always ἐν τῷ νῦν, **then** the flying arrow would be motionless.

T2$_8$: **If** everything **were** always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, since** what moves is always ἐν τῷ νῦν, **then** the flying arrow would be motionless.

T2$_9$: **If** everything **were** always either at rest or in motion when it is κατὰ τὸ ἴσον, **then, if** what moves **were** always ἐν τῷ νῦν, **then** the flying arrow would be motionless.

The point of this classification is to enable translators to make clear, by choosing from among the nine variants, their considered judgment concerning the modal status of each of Zeno's premisses or, more precisely, what they think Aristotle thought that Zeno had intended the modal status of each premiss to be. Although determination of the modal status of the premisses is not an end in itself, it will prove to be an important piece

of a mosaic, which, when combined with other pieces, will ultimately yield an authentic and defensible reconstruction of the *Flying Arrow*.

2. *The understanding of the phrases* κατὰ τὸ ἴσον *and* ἐν τῷ νῦν

As for the phrase κατὰ τὸ ἴσον, which is peculiarly Aristotelian,[17] its literal transcription is 'against what is equal'. With the exception of Prantl and Morpurgo who respectively translate it as 'behaves in the same way' and 'remains in the same state',[18] all commentators gloss '[is] against what is equal' by 'occupies an equal space' and understand this to be elliptical for 'occupies a space equal to its own volume/dimensions'.[19]

A lesser consensus attends translations of the phrase ἐν τῷ νῦν. One point on which interpreters disagree is whether this expression is to be understood in accordance with Aristotle's terminology, in which case it should be translated as 'at an instant' (past, present or future), or whether it should be given the meaning it had in Zeno's time, in which case it should be translated as 'in the now' (in the present). There are also notable differences in the way in which representatives of these two approaches understand 'instant' and 'the present'. Some think that 'instant' should be taken as extensionless because it is Aristotle's technical term (Guthrie, Vlastos, Barnes),[20] while others think that 'instant' should be given some atomic duration because they believe the validity of the *Flying Arrow* argument rests upon the presupposition that time consists of extended but indivisible instants (Tannery, Brochard, Lee, Cornford, Raven).[21] There is also a third view propounded by Owen according to which it is immaterial whether 'instant' is taken as durationless (extensionless) or as having atomic duration (atomic extension),

since Owen deems it sufficient for the validity of the argument that the arrow cannot move 'at an instant'.[22] As for the 'presentists' (our label for representatives of the second approach), they can be divided into three analogous camps. Thus, Calogero understands νῦν as the eternal present, since for him pastness and futurity have no reality.[23] By contrast, Le Poidevin thinks that 'the present' must be extensionless, for he believes that if it were not, it would contain both earlier and later times, something that would contradict the very concept of presentness.[24] Finally, there are those who think that it is immaterial in which way one understands 'the present', because they think it sufficient for the validity of the argument that the arrow cannot move 'in the present' (Lear, Schofield).[25]

The two phrases κατὰ τὸ ἴσον and ἐν τῷ νῦν encapsulate important parts of the argument. Attempts to link them to each other generate distinct interpretations that often include textual changes. These textual modifications represent the third item over which scholars differ, as the next section will make manifest.

3. *Textual changes*

In Zeller's widely accepted emendation of the *Flying Arrow* the words ἢ κινεῖται in the first proposition are omitted[26] in order to turn the phrase κατὰ τὸ ἴσον into a *definition of being at rest* (εἰ γὰρ ἀεί … ἠρεμεῖ πᾶν ὅταν ᾖ κατὰ τὸ ἴσον).[27] Some support for this emendation is found in Themistius' paraphrase of the *Flying Arrow*,[28] despite the fact that the omitted words are present in all the pertinent manuscripts of the *Physics* as well as in both Simplicius' paraphrase[29] and Philoponus' paraphrase[30] of the argument.

Nevertheless, many later interpreters (e.g., Renouvier, Burnet, Ross, Guthrie, Vlastos, Barnes, Schofield, Lear, Faris and others) [31] have followed Zeller's lead.

Because the just-mentioned omission lacks textual justification, some scholars have proposed another emendation in order to make the related point that 'nothing is in motion when it is κατὰ τὸ ἴσον'. According to this emendation, some words have supposedly dropped out of the original text[32] owing to *haplography* (e.g., εἰ γὰρ ἀεί … ἠρεμεῖ πᾶν ἢ κινεῖται καὶ μὴ κινεῖται ὅταν ᾖ κατά τὸ ἴσον).[33] But there is also no textual justfication for this emendation, since both Simplicius and Philoponus, in their paraphrases of the *Flying Arrow*, clearly relate κατὰ τὸ ἴσον both to the state of motion and to the state of rest (see notes 29 and 30).

Once they had turned the first premiss into a definition of rest or a statement that everything is at rest when it is κατὰ τὸ ἴσον (due to the fact that 'nothing is in motion when it is κατὰ τὸ ἴσον'), the above-mentioned interpreters tried to get the conclusion that the arrow is motionless by interpreting the second premiss to mean that 'what moves is always κατὰ τὸ ἴσον when it is ἐν τῷ νῦν'. This is why Zeller adds another κατὰ τὸ ἴσον after ἐν τῷ νῦν, again without any textual justification.[34] The same objection applies to the *explanation* Diels inserts after the second premiss (πᾶν δὲ κατὰ τὸ ἴσον ἐν τῷ νῦν) Ross, however, tries to reach the same goal without changing the second premiss by suggesting that the emphatic position of ἔστι *indicates* that what is said in the first premiss *is* always so when the moving object is in a moment.[35] The difficulty with Ross' proposal is that there are several examples in the *Physics* of sentences of the same structure where the second premiss begins with a verb, either 'to be' or some other verb, whose function simply cannot be explained in the way Ross suggests.[36]

However plausible the reasons for making textual changes to *The Flying Arrow* might appear, one must not lose sight of the fact that they have no textual warrant. For precisely this reason some scholars have insisted on the retention or preservation of the original text proposed by Bekker, tolerating appeal to tacit premisses only for its interpretation.

4. *The invocation of tacit premisses*

The French scholar Hamelin accepts Bekker's text of the *Flying Arrow* but contends that Aristotle does not cite Zeno's original argument in full.[37] According to Hamelin, Aristotle explicitly states only those propositions that he wishes to deny, namely:

1. A body that occupies a place equal to itself must be either at rest or in motion.

2. A body is always at an instant.

3. The flying arrow is motionless.

So, Hamelin thinks that Aristotle omits propositions that he takes to be unnecessary for his argumentation. In Hamelin's view, this alleged practice of Aristotle explains why his *Flying Arrow* appears incomplete. In order to render the argument complete, Hamelin invokes two allegedly tacit premisses:

1a. A body in a proper place is not in motion (allegedly implied by Aristotle in $239^{a}23$-$239^{b}4$).

2a. Time is composed of instants only (found in Aristotle's comment on the argument in $239^{b}7$-9).[38]

Interestingly, insertion of these two alleged tacit premisses leads to the same interpretation as the one obtained by the aforementioned textual changes.[39]

It is evident from our discussion thus far that all scholars, despite their many differences, agree that the *Flying Arrow* as presented by Aristotle needs emendation or supplementation. We agree, but unlike them we think the required supplementary material is to be found elsewhere in Zeno's own doctrines.

5. *The relation of Aristotle's Flying Arrow to DK 29 B 4*

Two doxographers, Diogenes Laertius (*Vitae philos.* IX 72) and Epiphanius (*Adv. haer*. III 11), quote an argument that they attribute to Zeno and whose briefer form, cited by Diogenes, is included among the four fragments of Zeno's teaching that Diels considers original (DK 29 B 4): τὸ κινούμενον οὔτ᾽ ἐν ᾧ ἔστι τόπῳ κινεῖται οὔτ᾽ ἐν ᾧ μὴ ἔστι ('The moving thing moves neither in the place in which it is nor in the place in which it is not').[40] In his *History of Greek Philosophy* Guthrie remarks that this fragment 'is very similar to the paradox of the flying arrow and may be only *a condensation of it*' [*our italics*].[41] Taylor, in his 1934 translation of Plato's *Parmenides*, was the first to recognize this similarity.[42] In 1942 Fränkel suggested a connection between B 4 and the *Flying Arrow*, but failed to go into details.[43] The same holds for Guthrie who interprets Aristotle's *Flying Arrow* without any reference to its 'condensed version', and for Calogero who simply gives credit to all the interpreters who recognized the connection between B 4 and the *Flying Arrow*.[44]

The first scholar who ventured a reconstruction of the *Flying Arrow* based jointly on Aristotle's text *and* on the closely related fragment DK 29 B 4 was Gregory Vlastos.

In his 'Note on Zeno's Flying Arrow' Vlastos offers, in our opinion, compelling reasons for accepting B 4 as an authentic statement of Zeno's and for attaching it to Aristotle's *Flying Arrow*. Since we agree with Vlastos and indeed base our own reconstruction on the same textual relationship, we now enumerate the evidence for Vlastos' position:

1. The ascription of B 4 to Zeno has the backing of both Diogenes Laertius and Epiphanius.

2. Diogenes Laertius, in a passage where he appears to be relying upon a trustworthy source, attributes the fragment to Zeno.

3. It is true that Sextus three times associates B 4 with the name of Diodorus Cronus (*Pyrrh. hyp*. II 245 and III 71; *Adv. math*. X 86-9) and never mentions its Zenonian authorship. But this does not mean that Diodorus was its originator, for Sextus is silent concerning the authorship of another unquestionably Aristotelian argument used by Diodorus in the same connection (*Adv. math*. X 89, 5-7). Moreover, as Fränkel observed, in *Adv. math.* X 87, Sextus remarks that Diodorus was proposing a 'current' argument.[45]

4. Anyone who accepts the authenticity of DK 29 B 4 must look upon it as part of a whole formed with Aristotle's *Flying Arrow*, because Aristotle himself says explicitly that 'there are four arguments by Zeno against motion' (*Phys*. 239^{b}10) so one could hardly claim that DK 29 B 4 represents a fifth. (No one would argue that DK 29 B 4 is related to any of Zeno's three other arguments against motion.)

Moreover, there is a consensus among the scholars who deal with this question (even if they doubt the authenticity of B 4) that Aristotle's *Flying Arrow* and DK 29 B 4 are similar in both content and point.[46]

In the revised edition of Kirk's and Raven's *The Presocratic Philosophers*, Schofield reiterates Vlastos' idea 'that the argument of Aristotle's Flying Arrow probably formed the first *limb* of the antinomy ascribed to Zeno in DL IX 72 (DK 29 B 4) and later borrowed by Diodorus Cronus (Sextus, *Adv. math*. X 87)' [*our italics*].[47] Schofield does not, however, mention Vlastos' theory that the role of Aristotle's *Flying Arrow* was to 'defend' the claim in DK 29 B 4 that 'the moving thing does not move in the place in which it is'. As for the complementary claim that 'the moving thing does not move in the place in which it is not', Vlastos believes that Zeno might have also provided an explanation of it, but not in the form of an argument, a contention we disagree with as will become evident below.[48]

Unfortunately, although remarked upon by many prominent scholars, the important connection between Aristotle's *Flying Arrow* and DK 29 B 4 has been explored only superficially, and never by using B 4 to throw light on Zeno's *Flying Arrow* argument in general or on Aristotle's *Flying Arrow* in particular.

Part II. *Two proposed reconstructions of the Flying Arrow argument*

Scholarly disagreement over the five points discussed above notwithstanding, one finds in the literature only two materially different reconstructions of Zeno's *Flying Arrow*. Surprisingly, considerations of logical structure and modal status (discussed in **I**.1) as well as considerations of the connection between the Aristotelian and Zenonian texts (discussed in **I**.5) have had virtually no impact on either of these two reconstructions, which we now proceed to examine.

1. *The Standard Reconstruction*

We call the first reconstruction *standard* because it is endorsed by a large majority of scholars, independently of the reasons they give to justify it. This reconstruction, in minute detail, goes as follows:

a) Everything is at rest whenever it occupies a space equal to itself.

b) At every instant, the flying arrow occupies a space equal to itself.

c) So, at every instant, the flying arrow is at rest.

d) The flying arrow is always at an instant, since time consists only of instants.

e) So, the flying arrow is always at rest during its alleged motion.

f) If the flying arrow is always at rest during its alleged motion, it is motionless throughout the duration of its alleged motion.

g) So, the flying arrow is motionless throughout the duration of its alleged motion.

h) If the flying arrow is motionless throughout the duration of its alleged motion, it does not change position.

i) So, the flying arrow does not change position, i.e., the flying arrow is motionless □ *tout court*.

According to this standard reconstruction, the flying arrow never changes position because it occupies a space equal to itself at every instant (whether instants are extensionless or not) and the flying arrow is supposed to be always at an instant (of a series of instants).

2. *The Presentists' Reconstruction*

As mentioned in **I.**2, translation of ἐν τῷ νῦν as 'in the now' or 'in the present' has been endorsed by Calogero, Lear, Schofield and Le Poidevin. For them, Zeno's conclusion follows immediately from the fact that the flying arrow is always *in the present* in which it does not move. Because the flying arrow is always *in the now* in which it occupies a space equal to itself, i.e., because it is always at rest, it follows immediately that the flying arrow is *motionless*.

We do not think that νῦν should be understood in this way. Were we forced to choose between the two foregoing reconstructions of the *Flying Arrow*, we would reluctantly embrace the *standard* reconstruction. Fortunately, this choice is not forced on us. Because Zeno's original argument is more complex and subtle than has been recognized, a third and more attractive alternative presents itself, as we shall now show.

Part III. *A new reconstruction of Zeno's Flying Arrow argument*

1. *The first premiss: a new understanding of the* κατὰ-τὸ-ἴσον *condition*

As we have seen above, the main source of the uneasiness of the *standard interpreters* of Aristotle's argument is the fact that in Bekker's edition of the *Physics* the κατὰ-τὸ-ἴσον condition, which says that the body under consideration occupies a space equal to itself, applies to the body when at rest as well as to the body when in motion (ἠρεμεῖ … ἢ κινεῖται). The reason for all the aforementioned textual changes is the assumption that a body can be only at rest, and never in motion, if it occupies a space

equal to itself. At the same time, as we have already emphasized, there are no philologico-historical grounds for questioning the authenticity of Bekker's edition of Aristotle's text.

There is, we claim, a simple straightforward way to resolve the conflict between the philologico-historical considerations and the logic of the argument. This way does not force one to translate the κατὰ-τὸ-ἴσον phrase in some non-standard fashion. It demands only that one recognize that the κατὰ-τὸ-ἴσον condition *together with* the either-at-rest-or-in-motion condition *can be satisfied*, and indeed *are jointly satisfied*, by bodies that are rigid during the time of consideration, and only by such bodies. It is perfectly natural to say that the fact that, during or at some time, a body constantly occupies a space equal to itself means that, during or at that time, the body *changes neither its shape nor its volume*. Furthermore, it is an entrenched commonsense belief that a body like an arrow retains its shape and volume independently of whether it moves or is at rest. Such bodies are commonly said to be *rigid*.

So, we can say that the first premiss implicitly introduces a rigid body as the protagonist of the argument, a protagonist that will be explicitly identified as the flying arrow only in the conclusion. But the fact that the argument deals with πᾶν ... ὅταν ᾖ κατὰ τὸ ἴσον ('everything *when* it is rigid') and not πάντα ὅσα κατὰ τὸ ἴσον ('everything *that* is rigid') shows that it does not refer only to things that are rigid *in se* but that it refers equally to things that do not change their shape or volume *per accidens* at or during the time of consideration. This observation makes it evident that the adverb ἀεί ('always') at the beginning of the first premiss is used to emphasize the fact that the body under consideration is *constantly* rigid at or throughout the entire time of consideration.

This means that Zeno's argument is about any body at any time or any stretch of time *when* it does not change its shape or volume, be it an arrow before it has been bent or burnt, or a snowball before it has begun to melt.

Even from a purely historical point of view, the temporal restriction in the first premiss is not irrelevant, because the Ionian physicists spoke of bodies that move by changing their shape and volume (namely: water, air, and fire).[49] Due to the great diversity of uses of κίνησις in Zeno's time, and in view of his objective to argue against a particular kind of κίνησις, viz., *locomotion*, it was necessary for Zeno to restrict the domain of pertinent bodies to those that, like arrows, do not change their shape or volume, at least not during the time of consideration. Although it is plausible to suppose that Zeno's ultimate aim was to deny the possibility of every kind of change, and *a fortiori* of every kind of motion (κίνησις), it is a *fact* that his Flying Arrow argument rejects or disallows a *particular* kind or type of motion (κίνησις, viz., locomotion of rigid bodies). Since motion from place to place of rigid bodies is the paradigm case of locomotion, Zeno undoubtedly believed that to discredit this kind of motion was *ipso facto* to discredit motion generally.

Finally, although in our view the role of the first premiss is to implicitly define the protagonist of the argument, the very formulation of this premiss also implies that there is no third state in which a body can be, i.e., no state other than motion or rest (the only states mentioned in the argument). It would be anachronistic to suppose that Zeno entertained the possibility of a state alternative to motion and rest, for such a *tertium quid* makes its first appearance in Aristotle's *Phys*. 239^{b}1-2.

Other advantages of our reinterpretation of the κατὰ-τὸ-ἴσον clause will become evident when we advance additional reasons for our interpretation, but even at this stage the reader will recognize that our interpretation has the significant advantage of preserving the wording of the *Flying Arrow* argument as found in Bekker's edition of the *Physics*.

We turn now to the issue of the modal status of the first premiss. From **I**.3-4 it is clear that all the aforementioned interpreters, whether they alter the text in Bekker's edition of the *Physics* or not, whether they take the first premiss to be a definition of rest (those who omit ἢ κινεῖται) or as a declaration that everything is at rest when it is κατὰ τὸ ἴσον (those who insert καὶ μὴ κινεῖται / οὐδὲν δὲ κινεῖται or who add tacit premisses), understand the first premiss to be something that Zeno accepted as true. According to our new interpretation, this first premiss has nothing to do with definitions of rest or with the proposition that everything is at rest when it is κατὰ τὸ ἴσον. Nevertheless, we too understand this premiss to be something that Zeno accepted as true, indeed as quasi-analytically true, since there are no grounds whatsoever for thinking that Zeno conceived of any state alternative to motion and rest (i.e., alternative to motion and non-motion), and since all that is asserted in this premiss is the virtual tautology that the rigid body is either in motion or at rest**,** i.e., that it is either in motion or not in motion. So, in accordance with the difference between 'since' and 'if' (*see* **I**.1 above), it is incontestable that the first premiss should be introduced by 'since'.

2. *The second premiss: its modal status and its relation to DK 29 B 4*

We note first that our understanding of Zeno's second premiss is based on the standard view that ἐν τῷ νῦν should be translated as 'at an instant' (see **I**.2). We concur with this view because it harmonizes with all uses of τὸ νῦν by Aristotle, independently of whether τὸ νῦν stands for a *durationless instant* or, in controversial contexts (such as 231^{b}18ff.), for an *atomic quantum of duration*.

Though we accept the standard interpretation of ἐν τῷ νῦν, we reject the standard account of the modal status of the second premiss of the *Flying Arrow*. With the exception of Vlastos, all the scholars mentioned in Part **I**, whether in the role of translators or in that of commentators and interpreters, believe that Aristotle took this premiss to be, not just something assumed by Zeno for the sake of argument, but something that Zeno accepted as true. Now, it is indisputable that all these scholars (independently of whether they alter Bekker's text or not) understand the first premiss in this same way, i.e., as something Zeno accepted as true. So, with respect to the modal status of the premisses, Cornford's transparent translation with 'since' at the front or head of both premisses can serve as the representative of all the aforementioned interpretations.

One might wonder about the reasons for the general consensus concerning the modal status of the second premiss as something that Zeno took for granted as opposed to something he merely supposed *arguendo*, so a brief historical excursion is in order. The tradition behind this consensus may have originated with Zeller, the first great scholar who re-examined Aristotle's presentation of the *Flying Arrow* and (as mentioned in **I**.3) proposed textual changes to Bekker's edition. It was also Zeller who translated the

second premiss as '*Nun aber* ist der fliegende Pfeil ...' [*our italics*]. It is evident from his translation that Zeller understood this premiss to be a claim of Zeno's and not merely an assumption made by Zeno. It is no wonder, then, that Zeller took Aristotle's rejection of the second premiss (*Phys*. 239^{b}7-9) to be a refutation of something that Zeno had himself actually espoused. Those who have followed Zeller come to the same conclusion by reasoning in the opposite direction: they take Aristotle's rejection of the second premiss to be grounds for taking it to express something that Zeno believed.[50] So, let us now take a closer look at this argumentation.

Aristotle opines that Zeno's argument would be sound if time were composed of indivisible instants. Aristotle's opinion is correct, and would remain correct even if the qualification 'indivisible' (atomic) were omitted. In his only other mention of the *Flying Arrow* (239^{b}30-33) Aristotle explicitly says that the argument would be sound[51] only 'if we took time to consist of instants'.[52] This remark, however, does not indicate whether the second premiss is something Zeno actually claims to be true or is just a supposition from which Zeno's conclusion follows. So, because Aristotle's rejection of this second premiss is neutral between an attribution to Zeno of a false claim or a false supposition, Aristotle's comment fails to determine the modal status of the second premiss.

Contrary to the general consensus, we believe there are good independent reasons, ones not mentioned even by Vlastos with whom we agree to this point, that favor an understanding of the second premiss as a mere argumentative assumption on Zeno's part.

Of the aforementioned reasons for taking the second premiss to be an assumption by Zeno, we are now going to discuss one having to do with the neglected fragment DK 29 B 4 (see **I**.5). If 'being always at an instant' is for a moving body just a possibility

introduced *arguendo* as an additional condition in *Physics* $239^{b}5$-7, one would expect that Zeno also dealt with its alternative. And if we accept Guthrie's suggestion that DK 29 B 4 is 'a condensation' of the *Flying Arrow*, then it is clear that the whole argument consists of two limbs or branches, as stressed by Schofield (see **I**.5 above), since what is said about the moving body in DK 29 B 4 is that it can move '*neither* in the place in which it is *nor* in the place in which it is not' [*our italics*].

We will explain in detail the branching of the argument and discuss its second limb later. For now it will suffice to say that Aristotle's *Flying Arrow*, which considers the case of a body that is always at some instant or other, constitutes the first limb, whereas the second limb is found in DK 29 B 4 where Zeno considers a body that is at least sometimes in a time interval. At this stage, it is important to realize that the linkage between Aristotle's *Flying Arrow* and DK 29 B 4, a connection noted by many prominent scholars, strongly supports our construal of the modal status of the second premiss as an assumption *arguendo*. When this same linkage is viewed from the opposite direction, our reading of the second premiss together with our new understanding of the κατὰ-τὸ-ἴσον condition will lead to a correct account of the second limb of the argument. The novelty of our interpretation, therefore, consists *inter alia* in our suggestion—for the moment only tentative—that the proper reading of Aristotle's *Flying Arrow* should be $T2_4$ or $T2_8$ (see **I**.1), contrary to all the aforementioned interpretations of the argument except that of Gregory Vlastos, who unfortunately failed to develop this idea.

Given our understanding of the modal status of the second premiss, one more thing remains to be clarified. If the second premiss states a special condition, then the 'always' occuring in it differs referentially from the 'always' occuring in the first

premiss. The 'always' of the first premiss leaves open whether its domain embraces instants or intervals. By contrast, in Aristotle's *Flying Arrow* the domain of the 'always' of the second premiss is unequivocally restricted to the set of instants at which the allegedly moving body is during its supposed motion.

3. *The conclusion of the argument in its first branch*

We turn now to a discussion of the validity of the *Flying Arrow* along the first of the two limbs we have envisioned for the original argument, i.e., the limb presented by Aristotle in *Physics* 239^{b}5-7.

First of all, in spite of the fact that our reinterpretation of the κατὰ-τὸ-ἴσον condition (given in **III**.1 above) allows the (rigid) body to occupy a place equal to itself independently of whether it is at rest or in motion, i.e. regardless of its being at an instant or in a time interval, it can certainly be taken as true that the body is motionless if it is at an instant (as supposed by the second premiss). So, our interpretation retains everything the standard intepretation needs in order to explain how Zeno reaches the conclusion that the flying arrow remains motionless. Nevertheless, one might object that, though our interpretation of the first premiss retains everything needed to reach this conclusion in the standard way, it contains more than what is necessary, for the fact that at an instant a body occupies a space equal to itself holds for rigid and non-rigid bodies alike. Our response is that the significance of our understanding of the κατὰ-τὸ-ἴσον condition will become apparent only in the second branch of the argument, so that Zeno's conclusion will hold for the body independently of whether it is always at an instant or is at least

sometimes in a time interval. And it will turn out that this body will have to be rigid, which after all it is, since it is explicitly said to be an arrow.

Though our reinterpretation of the first premiss contains everything needed for the standard interpretation, we strongly doubt that Zeno intended to say that the flying arrow does not change position just because it is motionless at every instant, as the standard interpretation contends. This is important, as we shall shortly see, because one could say, as Russell explicitly did and Diodorus Cronus ingeniously anticipated,[53] that the arrow can change its position by simply being in different positions at different instants. So, the crucial thing, we believe, concerns the way in which τὸ νῦν is to be understood in the argument.

In **III**.2 we rejected the presentists' position but left open whether 'instant' in the second premiss should be understood as durationless or as an atomic quantum of duration. We agree completely with Vlastos on the following three points.[54] First, we agree that in showing why the argument is unsound Aristotle uses τὸ νῦν as an atomic quantum of duration, for otherwise the qualification of τὸ νῦν as 'indivisible' would be superfluous (*Phys*. 239^{b}8-9) (and, as is well-known, Aristotle himself thinks that time does not consist of durationless instants). Second, we also agree that Aristotle's observation does not mean that Zeno himself, in the argument, considered atomic quanta of duration, for, as Vlastos argues convincingly, there is no trace of such a notion in Zeno's time. Third and finally, we agree with Vlastos that the manner of Aristotle's rejection of the soundness of the argument does not imply that Aristotle thought that Zeno himself had in mind such entities as atomic time stretches. In his refutation of the *Flying Arrow* Aristotle does not say that Zeno is wrong for *claiming* that time is

composed of instants, but only that Zeno's argument *would fail to be sound* without the *assumption* that time is composed of instants. So, Aristotle's comment 'Zeno argues fallaciously' refers to *the logical character of Zeno's argument* and not to something that Zeno *believed*.

If we are right about all this, then Zeno's objective in the first limb of the argument (the only branch discussed by Aristotle) was not to show that the flying arrow does not change position *tout court*, but only that it does not change position by virtue of being in different positions at different instants. We are now going to examine the resources that Zeno himself had at his disposal to show that the flying arrow does not change position in the aforementioned way, something Vlastos himself neglected to do. Thereafter, we shall turn to the second limb and investigate the alternative to the second premiss that was envisioned by Zeno himself.

When considering in DK 29 B 2 that which has no magnitude as a possible candidate for a constituent of spatial magnitude, Zeno rejects this possibility on the grounds that such an entity contributes nothing to a magnitude (because it makes nothing greater when added and nothing smaller when subtracted). He could surely have reasoned in the same way about whether instants could be constituents of time. If he had done this—and it is altogether natural to suppose that he had—he could equally well have taken into account the *static (cinematographic) theory of motion* and rejected it on the analogous grounds that time does not consist of durationless instants. But let us suppose, *arguendo*, that Zeno had not done this. Even then he could have concluded in the first limb of the *Flying Arrow* that the arrow does not change position, not because it is at rest at every instant (as contended by the standard interpretation), but because it would be in a

position or place (at some other instant) that is distant from the position or place in which it already is (at some given instant) only if it could have been in all the intermediate positions (at all the instants between the instant at which it is in the given position and the instant at which it would be in the hypothesized distant position). According to this analysis, the impossibility of the arrow's moving by virtue of being in distinct positions at distinct instants rests completely on what is envisioned in DK 29 B 2: the intermediate positions cannot be exhausted in this way. So, even if Zeno did not consider the problem of the constitution of time (which is unlikely, since the analogy between space and time is obvious), he could still have correctly inferred that the flying arrow does not change its position on the basis of what he had concluded in DK 29 B 2.

Interpreted in such a way, Zeno's reasoning in the first limb of the argument is more subtle than any of the extant interpretations make it out to be, for the point is not that the flying arrow does not change position because it is at rest at every instant, but that there are independent reasons why it is not possible for the flying arrow to change position in such a way. These reasons are encapsulated in what Aristotle calls 'Zeno's Axiom' (*Metaph*. $1001^{b}7$ff.).

There are two ways to understand the proposition that Aristotle refers to as 'Zeno's Axiom', but only one of these ways is uncontested and contained in DK 29 B 2, viz., as the claim that 'that which has no magnitude cannot be a consitituent of a multitude'. In his comment on this claim, Aristotle says that in this same sense a plane cannot be composed of lines and a body (i.e., a solid) cannot be composed of planes. Aristotle clearly means that 'Zeno's Axiom', in its generalized form, claims that an entity of a higher dimension cannot be built up out of entities of a lower dimension. What can

be contested is the claim that Zeno concluded from his eponymous axiom that points (and thus lines and planes, too) are absolutely nothing. It is important to note that Aristotle himself says only that Zeno 'does not say that it [a point] is a being' (οὔ φησιν εἶναι τοῦτο τῶν ὄντων, *Metaph*. 1001^{b}9). In view of these facts, when we use the term 'Zeno's Axiom', we shall understand it solely in its uncontested and central meaning. As to the contested claim, one should note that in the *Flying Arrow* Zeno did not reject instants (τὰ νῦν) as non-existing (on the grounds that the instant has no duration), but expressly allowed for the possibility that the moving body is 'always at an instant' (ἀεὶ ἐν τῷ νῦν).

Given the uncontested and central meaning of *Zeno's Axiom*, it is quite natural to suppose that Zeno took this axiom to be a tacit premiss of the *Flying Arrow*. Unlike the tacit premisses proposed by the authors mentioned in **I**.4, this tacit premiss can be truly ascribed to Zeno (as an axiom that bears his name, after all) and makes the argument not only valid but also illuminating, for it explains *why* the flying arrow does not move if it is supposed to be 'always at an instant'. To derive his conclusion in this way Zeno did not have to consider atomic magnitudes of any kind, not even hypothetically. So, it turns out that in the first limb of the argument Zeno and Aristotle reached the *same* conclusion, viz., that the arrow does not move by virtue of being always at an instant, by *different* routes. Zeno appealed tacitly to the premiss that Aristotle calls *Zeno's Axiom*, whereas Aristotle first supposed and then rejected the proposition that time consists of atomic quanta of duration.

Let us settle finally the dispute concerning the translation of the *Physics* passage 239^{b}5-7. We have already argued in **III**.1 that the first premiss should be introduced by 'since' because it is analytically true that a rigid body (the arrow) does not change shape

or volume, independently of whether it is at rest or in motion (this would be the case even if the body could never move). Our foregoing argumentation makes clear that the second premiss should not be taken to be a *claim* made by Zeno, but we have left it open whether this premiss should be understood as a condition with nothing implied as to its fulfillment, which would result in translation T2$_4$, or as a counterfactual as the basis for an inference, which would result in translation T2$_8$ (see **I**.1). We will now argue for translation T2$_4$. Since we ourselves want to include *Zeno's Axiom* among the premisses of the argument, it might seem that we would prefer T2$_8$. But it is not Zeno's practice to base his arguments on suppositions that are naturally understood as counterfactuals, as in the famous example from Herodotus where εἰ + present indicative is properly translated by 'if the dead were to rise'.[55] To the contrary, whenever Zeno supposes something in an argument, it is always something that everyone must accept either *simpliciter* or at least *prima facie*.[56] Accordingly, the second premiss of the *Flying Arrow* should be understood to be something initially acceptable to the reader or listener, but which will later be dismissed as impossible (by appeal to an evident truth such as *Zeno's Axiom*). Therefore, in conformity with Vlastos' understanding of the modal status of the two premisses (though not with his acceptance of the textual changes in the first premiss and its resulting interpretation), we obtain the following translation of the *Flying Arrow* argument, expressed as an object-language conditional:

> *Since everything is always either at rest or in motion when it does not change shape or volume, then, if what moves is always at an instant, the flying arrow is motionless.*

So, ironically for the traditionalists, Zeno and Aristotle turn out to have agreed (according to our interpretation) that Aristotle's *Flying Arrow* is a *valid* argument and that it would be a *sound* argument if its second premiss were true. But given that Aristotle did not treat this argument as branching, i.e., as if there were an alternative to the second premiss, it was for him unsound *simpliciter*. However, Zeno's *Flying Arrow* argument in its original form cannot be logically dismissed as unsound *before* examining the alternative to the second premiss, which introduces another possibility for the constitution of time.

4. *The second branch of the argument*

As we have just established, the conclusion of Aristotle's *Flying Arrow* is validly inferred both by Zeno and by Aristotle. However, the fact that the second premiss is incompatible with *Zeno's Axiom* (which is a tacit premiss) naturally leads one to consider alternatives to this premiss. Happily, we are in a position not only to argue *that* Zeno took an alternative into consideration, but even to reconstruct *the way in which* he did this.

As already stated above, the interpretation of DK 29 B 4 as a condensed version of the *Flying Arrow* clearly indicates that what is said in *Physics* $239^{b}5$-7 is just one of two branches of the argument. But as the way in which the argument is condensed in DK 29 B 4 is rather vague, we will now clarify its meaning with the help of the branch that has already been reconstructed and by comparing the whole argument whose reconstruction we are seeking with Zeno's arguments against plurality preserved in DK 29 B 1-2.

In the arguments against plurality (DK 29 B 1-2) Zeno deals with two possible candidates (both later discarded as inadequate) for constituents of a multitude. One candidate is the spatial analogue of τὸ νῦν (as argued in **III**.3), whereas the other is something having magnitude (DK 29 B 1). It is natural to suppose that Zeno makes the same distinction when he considers the time during which the allegedly moving body is supposed to move. The temporal analogue of a spatial magnitude is a time interval, so the alternative we are seeking to the second premiss is the proposition that a moving body is not 'always at an instant' but *at least sometimes is in a time interval*.

5. *A reconstruction of the second branch of the argument*

In the reconstruction of the second branch we shall follow the pattern already established in our reconstruction of the first branch. That is to say, we shall discuss each of its premisses separately in order to show how Zeno reaches the overall conclusion that the flying arrow remains motionless. The main problem in this reconstruction is posed by the formulation of the second premiss, since this is the only piece of the second branch that is not explicitly stated anywhere.

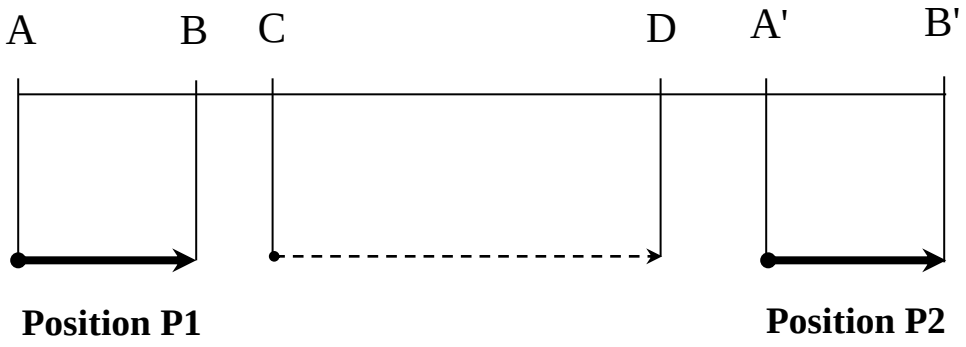
The first premiss of the second branch is to be understood in the same way in which it has been understood along the first branch (**III**.1), namely, as a general condition supposedly satisfied in the whole argument. So, again, we are dealing with a body that does not change its shape or volume, be it at rest or in motion, during the time of consideration, i.e., with a rigid body.

The second premiss of the second branch introduces a special condition that is assumed as true only in this branch of the argument. As we showed above (**III**.4), this

condition is the only alternative envisioned by Zeno to the assumption that the allegedly moving body is always at an instant, and it states that the body is at least sometimes in a time interval. The second premiss of the first branch has been symbolized by B, so let us represent this alternative premiss as B*.

When this second premiss is thus understood, its meaning is transparent: in order to move, a moving object requires some time, and since the flying arrow is supposed (in this branch) to be in a time interval, it can also be reasonably supposed that the arrow can move. The main question is how this *prima facie* acceptable possibility relates to the first premiss, which expresses a general condition upon the arrow as a rigid body.

Note that if the allegedly moving arrow were able to bridge or fill the gap between two designated positions of its supposed trajectory, then it would not be 'against something that is equal' in the time interval stretching between the two instants at which it is in these two designated positions, as well as in subintervals of this interval. That is to say, that which the arrow is against during the duration of its motion must be greater (longer) than the arrow itself. Were it otherwise, the flying arrow would have remained stationary. (In the diagram below, P1 and P2 represent positions of the allegedly moving arrow at distinct instants, so in the intervals AB and A'B' the arrow is against something equal to itself, whereas in the intervals AB' and CD it is against something greater than itself.)



This consideration shows that the first premiss, which expresses the general condition that the body under consideration is rigid, and the second premiss, which expresses the special condition that the body is in a time interval, are *incompatible* if the body is supposed to be in motion. At this stage of the reconstruction of the second branch of the *Flying Arrow* the importance of the general condition concerning rigidity expressed in the first premiss becomes crucial. The *gloss* of the κατὰ-τὸ-ἴσον condition as 'occupying a place equal to itself' is adequate to the first branch where it is argued that the body does not move by virtue of being always at an instant. In the second branch of Zeno's original argument the *literal* sense of the phrase 'is against what is equal' shows its importance, for, when it is in a time interval, the allegedly moving body is not against what is equal. We take this fact to be one of the main points in favor of our reconstruction of the *Flying Arrow*.

Let us now try to explain the meaning of the rather obscure and condensed version of the *Flying Arrow* found in DK 29 B 4. The phrases ἐν ᾧ ἔστι τόπῳ and ἐν ᾧ μὴ ἔστι (τόπῳ) mean literally 'in the place where (it) is' and 'in the place where (it) is not', respectively. Now, in position P1 as well as in position P2 (see the above diagram) the arrow is surely 'in the place where it is', but this does not hold, in an obvious sense, between these two positions, viz., when the arrow is in motion. In this latter case the arrow is *against something that is greater*, and it is likely that Zeno expressed this idea by his observation that the arrow would in this case have to be 'in the place where it is not'. So, if we remember that both κατὰ τὸ ἴσον and ἐν τῷ νῦν are Aristotle's words—but definitely not Zeno's words—it is plausible to think that Zeno expressed the

first and second premisses of the first branch in a condensed way, viz., through a single premiss asserting that the (allegedly) moving body moves by being 'in the place where (it) is', and then subsequently expressed the alternative by saying that the moving body moves by being 'in the place where (it) is not'.

In brief, our reconstruction not only coheres with but it even complements Aristotle's account of the first branch of the argument. It also makes sense of the otherwise strange-seeming phrases in DK 29 B 4 and even allows us to make a plausible conjecture about how Zeno himself formulated the premisses of the argument. So, though we agree with Barnes that it is hopeless to try to find the actual wording of Zeno's original argument on the basis of Aristotle's treatment of it,[57] we think that it is far from fruitless to try to do so if one also takes DK 29 B 4 into account.

6. G*eneral conclusion and schematic presentation of the whole argument*

The foregoing reconstruction of the *Flying Arrow* shows that the conclusion that the arrow cannot change position follows logically in each branch from the incompatibility of each branch's premisses. In the first branch the incompatibility is between its second premiss and the tacit premiss known as *Zeno's Axiom*, whereas in the second branch the incompatibility is between its first and second premisses. Thus, in our reconstruction, the *Flying Arrow* argument is an instance of what logicians call *proof by cases*. Although our reconstructed argument is valid, its *soundness* requires that the general condition (i.e. the first premiss) be true and that there be no third possibility (no third case) in addition to the two special conditions cited in the second premisses of the two branches. This is the place, therefore, to explain why the first premiss in Aristotle's

presentation of the *Flying Arrow* implicitly defines a rigid body (the arrow) to be the protagonist of the argument, in spite of the fact that the conclusion in the first branch holds for rigid and non-rigid bodies alike. In addition to the fact that our understanding of the κατὰ-τὸ-ἴσον condition was originally based on the desire to preserve the text as it stands in Bekker's edition of the *Physics*, it shows its full force only now when we consider the structure of the argument as a whole, viz., as a proof by cases. To show by cases that a body cannot move in either of two envisaged ways, one must refer to one and the same kind of body in each case. So, since the incompatibility of the premisses in the second branch turns on rigidity, the body has to be rigid in the first branch as well. The far-reaching significance of the general conclusion of the argument is not weakened by its restriction to bodies that are rigid at or during the time of consideration, since, as mentioned above, motion of rigid bodies from place to place is the paradigm of locomotion.

We will now investigate the logical structure of the argument as a whole. Remembering which propositions A, B, B* and C designate (see **I**.1 and **III**.5) and given that the first premiss along both branches is A, that B and B* are respectively the second premiss of the first branch and the second premiss of the second branch, and introducing Z for *Zeno's Axiom*, we get the following proof *ex hypotheses*:

(1) $\neg$ C, A, B, Z $\vdash$ $\neg$ (B & Z),

From (1) by *reductio ad absurdum*, we get

(2) A, B, Z $\vdash$ $\neg\neg$ C.

Similarly, from

(3) $\neg$ C, A, B* $\vdash$ $\neg$ (A & B*)

we get again by *reductio ad absurdum*

(4) A, B* ⊢ ¬ ¬ C.

Now, from (2) and (4), by elementary logic we get

(5) A, (B & Z) ∨ B* ⊢ ¬ ¬ C,

and if we take as undeniable both Z (because it can be treated as an axiom in the traditional sense of the word), as well as B ∨ B* (because there is no third possibility beyond B and B*), logic takes us from (5) to

(6) A ⊢ ¬ ¬ C,

and finally by double negation from (6) we obtain

(7) A ⊢ C.

If we remember that A means that the body under consideration does not change its shape or volume irrespective of whether it moves or is at rest, then (7) simply means that such a body (a rigid body) does not move.

The foregoing formal proof makes visible the macro-logical structure of Zeno's reconstructed reasoning and it can be formulated verbally as a branching syllogism. But before thus presenting it, let us notice that, according to our reconstruction of the argument, there is an implicit difference between *being in motion* and *changing position*, though it is difficult to believe that Zeno himself drew such a distinction. In the first branch the arrow is never in a *state of motion* because it is always at an instant, and this, according to Ross, was *all* that Zeno concluded.[58] According to the standard interpretation, this was *not everything* that Zeno concluded, but it was enough to enable him to establish conclusively that the flying arrow does not change position (see **II**.1). So, the standard interpretation also recognizes the implicit difference we are speaking

about. We disagree only with the standard interpretation's claim that Zeno leaped directly to the conclusion that the arrow does not change position, for, as argued in **III**.3, we believe Zeno invoked the principle that Aristotle calls *Zeno's Axiom* to reach this conclusion.

In the second branch, however, the arrow is first *allowed* to be in a *state of motion*, something that only later turns out to be impossible because of the incompatibility of the first and the second premisses. We shall make the foregoing implicit difference between changing position and being in motion explicit in our presentation of the syllogism. This difference will appear in the formulation of the conclusion of the first and the second branch, which will be denoted by C1 and C2, respectively.

(A) **Everything is always either at rest or in motion when it occupies a space equal to itself (is against what is equal).**

(B) **The arrow is always at an instant.**

(B*) **The arrow is at least sometimes in a time interval.**

(Z) **Time does not consist of instants. (Lines do not consist of points, etc.)**

(C1) **The arrow does not change position.** (C2) **The arrow does not move.**

(C) **The arrow does not move.**

Part IV. *Philosophical significance of the reconstructed Flying Arrow argument*

In our reconstructed *Flying Arrow*, Zeno's argument comes out *valid along both of its branches*. It follows that any theory of motion, no matter how modern or

sophisticated, must reject at least one of Zeno's premisses if it is to be coherent. The two historically dominant theories of motion—the *dynamic* theory and the *static* theory—can be obtained from the *Flying Arrow* simply by rejecting (as false or as not unqualifiedly true) one of the premisses necessary for reaching C1 or C2, respectively. Moreover, as we shall soon see, the creators of the static theory say explicitly that it is based on the rejection of *Zeno's Axiom*. It might appear, however, that the origin of the dynamic theory has nothing to do with the *Flying Arrow*, but we shall show that this is not so.

1. *The dynamic theory of motion*

It was Aristotle who created the theory of motion, later called *dynamic,* by using his richly-ramified theory of meaning, according to which one has to recognize πολλαχῶς λεγόμενα ('things said in different ways') as a fact and then deal with this fact by distinguishing *primary* and *secondary* meanings of what is said.[59]

Let us see how Aristotle's theory of meaning enables one to talk coherently of the motion of a rigid body. In *one sense*, it is true that even when in motion a rigid body is always κατὰ τὸ ἴσον, when κατὰ τὸ ἴσον is taken to mean that the body remains *as great as it is* (i.e., when it remains rigid). Moreover, this sense is the *primary* sense (πρῶτον) of κατὰ τὸ ἴσον when we speak of the motion of a *rigid* body. But this does not mean that a rigid body must be said to be κατὰ τὸ ἴσον when we speak of it *with respect to something else* and *not* simply with respect to its rigidity. When *in motion*, the rigid body is still said to be κατὰ τὸ ἴσον in the sense that it remains rigid, but, as moving, it is not then said to be κατὰ τὸ ἴσον in the sense of being against what is equal: 'And since whatever is in motion moves in a period of time and changes from one position to another, it is

impossible that the mobile should in its entirety be exactly over against any definite (stationary) thing (κατά τι) during the period occupied by this motion—occupied, that is to say, in the proper sense, not in the sense that the motion falls within some part of the period in question' (*Physics*, 239[a]25ff., Cornford's translation). So, Aristotle's point is that there is *no contradiction* in saying that a *moving rigid* body (a flying arrow, for example) *is* κατὰ τὸ ἴσον and *also not* κατὰ τὸ ἴσον, not only because it is κατὰ τὸ ἴσον as *rigid* and *not* κατὰ τὸ ἴσον as *moving*, but also because the phrase κατὰ τὸ ἴσον is itself used in *different ways and so has different senses* (πολλαχῶς λέγεται).[60]

The pragmatist maxim that there is no difference in meaning without a difference in reality challenges one to produce examples that illustrate the differences in meaning to which Aristotle appeals in his theory of motion. This challenge is easily met. If two identical rigid bodies move in tandem, they are, in spite of their motion, κατὰ τὸ ἴσον when viewed in relation to each other (as a consequence of their rigidity), while they are not κατὰ τὸ ἴσον when considered in relation to something else relative to which they both move (as a consequence of their being in motion). As for the *semantic priority* of the meanings of the phrase κατὰ τὸ ἴσον, there is in this imagined case no absolute fact of the matter. For each moving rigid body *is* κατὰ τὸ ἴσον in the *primary* sense if it is viewed relative to the other body, since it is then compared with another rigid body that moves in tandem with it. The situation is *reversed* if either rigid body is viewed as *moving*, because it is then compared in the first instance with *something else* relative to which it moves.[61]

Aristotle's theory of motion is based on his subtle new theory of meaning.[62] *Inter alia*, this theory implies rejection of the *unqualified generality* of what is stated in the first premiss of the *Flying Arrow* (as we have reconstructed it). The conclusion in the

second branch of the argument does not follow if one differentiates the two senses of κατὰ τὸ ἴσον, so one might be tempted to think that Aristotle arrived at his theory of motion simply by bringing his new theory of meaning to bear on Zeno's argument. But if this was Aristotle's actual procedure, why didn't he mention it?

The answer relates to the reason why Aristotle ignored the second branch of the *Flying Arrow*. Aristotle may not have possessed the original version of Zeno's argument, for if he was acquainted with the argument as a whole, then his neglect of its second branch is puzzling indeed. And even if Aristotle deliberately left the second branch out of *Physics* $239^{b}5$ff. (as irrelevant to what he discusses there), he could surely have profited from considering it in his discussion of motion in general. But whatever the reasons why Aristotle failed to mention Zeno when formulating his own dynamic theory of motion, it is a fact that in this context he gave less credit to Zeno than the latter deserved. For once the second branch is on the table, it becomes natural and easy to question the unqualified generality of the first premiss on the basis of the equivocation in the κατὰ-τὸ-ἴσον condition.

Our historical speculations concerning why Aristotle did not mention Zeno when formulating his dynamic theory of motion aside, the significance of our reconstruction of the *Flying Arrow* consists *inter alia* in showing that the dynamic theory can be obtained simply by restricting the unqualified generality of the κατὰ-τὸ-ἴσον condition.

2. *The static theory of motion*

Because the competing '*theorists* [...] either leave the ultimate elements of matter totally indeterminate, or [...] they assume them to be so-called *atoms* of *very small*, yet

not entirely vanishing space-content' the 'great struggle' among the followers of Aristotle and Epicurus so scandalized Georg Cantor that he was unwilling to let the issue go unresolved.[63] Following Weierstrass and Dedekind and boldly rejecting *Zeno's Axiom*, Cantor created the *point-based* conception of the continuum according to which a linearly ordered set of points *constitutes* a *continuum* if the set is *perfect* (*perfekt*) and *coherent* (*zusammenhängend*). Thus Cantor denied *Zeno's Axiom* by claiming that an infinite number of points constitute a line if each point of the set is an accumulation point of an infinite number of other points of the set, and each accumulation point of an infinite number of points of the set is itself a point of the set.[64]

In view of the reasonable and natural assumption that what holds for the structure of space (and matter) should also hold for time, Russell was quick to apply Cantor's theory of the continuum to time and thereby claim that the flying arrow can change its position simply by virtue of being in different positions at different instants.[65] But unlike Aristotle, when presenting this view Russell explicitly mentions Zeno. Russell acknowledges that the possibility of *defining* the motion of a body in terms of its being in different positions at different instants (as in his *static* theory of motion) is based on the rejection of *Zeno's Axiom*, a maneuver for which Russell credits Weierstrass.[66] According to the static theory of motion, then, it is possible to *change position* without ever being in a *state of motion*.

The task of evaluating the dynamic and static theories of motion lies far outside the scope of this paper,[67] but we hope that we have succeeded in showing that both theories have Zeno's *Flying Arrow* as their common origin.[68]

ENDNOTES

[1] I. Bekker, *Aristotelis opera*, edidit Academia Regia Borussica, vol. i (Berolini, 1831).

[2] We adopt Alonzo Church's canonical spelling 'premiss' for a logical assumption. See Alonzo Church, *Introduction to Mathematical Logic*, vol. i (Princeton, 1956), p. 1.

[3] We have interpreted the *Flying Arrow* passage as indirect discourse, but we note that the difference between direct and indirect discourse is, as here, not ordinarily reflected by a change in mood in Greek.

[4] For the sake of clarity, we have made explicit the implicit particle 'then' that in English introduces the consequent of the conditional or the conclusion of the argument.

[5] See, e.g., Barnes' comment paraphrased in the last paragraph of **III**.5. See also endnote 17 below.

[6] We use the turnstile symbol ⊢ to indicate or represent an inference from the formulas or sentences on the left of it to the formula or sentence on its rightside. When there is no formula or sentence on the leftside as in ⊢ C, one may take it to indicate or represent either the unqualified assertion of C or the inference to C from no premisses. When no formulas or sentences occur on its leftside, the turnstile is often deleted. Thus we can understand the form (A & B) ⇒ C of translation (1) to contain an initial implicit turnstile and so to indicate or represent either the unqualified assertion of (A & B) ⇒ C or the inference to (A & B) ⇒ C from no premisses.

[7] Cf. G. S. Kirk, J. E. Raven and M. Schofield, *The Presocratic Philosophers* [*Presocratic Philosophers*], (Cambridge, 1999; 2nd edn.), 273; H. D. P. Lee, *Zeno of Elea* [*Zeno*] (Amsterdam, 1967; 2nd edn.), 52; J. Barnes, *The Presocratic Philosophers* [*Presocratic Philosophers*], vol. i (London, 1979), 276; J. Burnet, *Early Greek Philosophy* [*EGP*] (London, 1908; 4th edn.), 367.

[8] See G. Calogero, *Studien über den Eleatismus* [*Eleatismus*] (Darmstadt, 1970), 141; J. Barnes, *The Complete Works of Aristotle* [*Complete Works*] (Princeton, 1991), vol. i; F. A. Shamsi, 'A Note on Aristotle, *Physics* 239^{b}5-7: What Exactly Was Zeno's Argument of the Arrow?' ['What Exactly Was Zeno's Argument of the Arrow?'], *Ancient Philosophy* 14 (1994), 51-71 at 52.

[9] H. G. Liddel, R. Scott and H. S. Jones, *Greek-English Lexicon* [*LSJ*], s.v. εἰ i.

[10] *Ibid*., *loc. cit*. vi.

[11] Cf. R. Kühner, *Ausführliche Grammatik der Griechischen Sprache*, Zweiter Teil, Satzlehre, Zweiter Band (Hannover und Leipzig, 1904; 3rd edn.), 466.

[12] F. M. Cornford, *Aristotle. The Physics* [*Physics*] (2 vols.; Cambridge, 1968-70; 4th edn.), vol. ii.

[13] In speaking of translation from English into English, we follow the lead of W.V.O. Quine who speaks of English-English translation manuals (cf. Chapter Two of his *Word and Object*, (Cambridge, Mass.: MIT Press and John Wiley & Sons, 1960). Same-language translation is no Pickwickian extension of the notion of translation. Not only is it central to some accounts of meaning but it poses the principal threat to Quine's thesis of the indeterminacy of translation (*ibid.*), which posits an inescapable multiplicity of radically inequivalent translation manuals, even for same-language translation (cf. G. J. Massey, 'The Indeterminacy of Translation: A Study in Philosophical Exegesis', *Philosophical Topics* 20 (1992), 317-343).

[14] The alternative choice, to represent Zeno's argument as an inference from premisses, would have equally serious drawbacks.

[15] Note that one of these antecedents is the antecedent of the whole formula, whereas the other is the antecedent of the consequent of the whole formula.

[16] Boldface type is used here to accentuate the modal status of the propositions that correspond to the premisses of the *Flying Arrow*.

[17] See G. Vlastos, 'A note on Zeno's Arrow' ['Note'], in R. E. Allen and D. J. Furley (eds.), *Studies in Presocratic Philosophy*, vol. ii, 184-201 (London, 1975) at 194, n. 2: 'I follow the usual translation of this peculiar phrase (cf. e.g. Burnet, 'when it occupies a space equal to itself'). I assume that if Zeno had used this expression his expansion of it would have been κατὰ τὸν ἴσον ἑαυτῷ τόπον, since τόπος would have been the only word he is likely to have used in this connection: cf. τόπον ἀλλάσσειν in Parmenides, frag. 8, 41 (Diels–Kranz). However, Zeno is more likely to have written ἐν τῷ ἴσῳ ἑαυτῷ τόπῳ for the context suggests strongly that the construction with κατά is Aristotelian. Aristotle starts talking of a mobile being κατά τι as far back as 239 A 25, using the phrase again at 30, 34, 35 and (twice) at 239 B 3. Uninterested in conserving the mention of τόπος in his summary of the puzzle (τόπος plays no role in the analysis of its reasoning presupposed by his refutation), it would be natural for him to change ἐν τῷ ἴσῳ ἑαυτῷ into κατὰ τὸ ἴσον in conformity with his 6 uses of κατά τι in the preceding 15 lines'.

[18] C. Prantl, *Aristoteles' acht Bücher Physik* (Leipzig, 1854), 322: 'wenn es in gleichmäßiger Weise sich verhälte'; E. Morpurgo, 'Intorno ai due ultimi ragionamenti di Zenone d' Elea contro il moto', *Giornale critico della filosofia italiana* 3 (1922), 209-225 at 214: 'permanere nella medesima condizione'. We supply English versions of their translations.

[19] Apropros this gloss, see Vlastos, 'Note', 194, n. 2 and Barnes, *Presocratic Philosophers*, 278. As for the translation 'occupies a space equal to itself', *cf.* Burnet, *EGP*, 367; H. Carteron, *Aristôte. Physique* [*Physique*], vol. i (Paris, 1961; 3rd. edn.), 60; V. Brochard, 'Les arguments de Zénon d'Elée contre mouvement' ['Les arguments de Zénon'], *Etudes de philosophie ancienne et de philosophie moderne* (1926), 3-14 at 6; Lee, *Zeno*, 53; W. K. C. Guthrie, *A History of Greek Philosophy* [History of Greek Philosophy], vol. ii (Cambridge, 1971; 3rd edn.), 93; V. C. Chappell, 'Time and Zeno's Arrow' ['Time'], *The Journal of Philosophy* 59/8 (1962), 197-213 at 198; Shamsi, 'What Exactly Was Zeno's Argument of the Arrow?', 52. 'Occupies a space equal to its own dimensions' translates Cornford, *Physics*, vol. ii, 180. 'Occupies a place just its own size' have Kirk, Raven and Schofield, *Presocratic Philosophers*, 273, and J. D. Lear, 'A Note on Zeno's Arrow' ['Note'], *Phronesis* 26 (1981), 91-104 at 91.

[20] Guthrie, *History of Greek Philosophy*, 93; Vlastos, 'Note', 187-190; Barnes, *Presocratic Philosophers*, 278ff.

[21] Having supposed that Zeno's arguments were directed against Pythagoreans according to whom time allegedly consists of indivisible quanta sometimes called *chronons*, French scholar Paul Tannery (P. Tannery, *Pour l' historie de la science Hellène* (Paris, 1930; 2nd edn.), 249ff.) was the first to conclude that the 'indivisible instants' assumed by Zeno's argument are atomic time stretches. Tannery was followed by a number of prominent scholars, such as Brochard, 'Les arguments de Zénon', 6, Lee, *Zeno*, 102-6, Cornford (F. M. Cornford, *Plato and Parmenides* (London, 1939), 58-9), Raven (J. E. Raven, *Pythagoreans and Eleatics* (Cambridge, 1948), 75) and others.

[22] G. E. L. Owen, 'Zeno and the Mathematicians', in R. E. Allen and D. J. Furley (eds.), *Studies in Presocratic Philosophy* (2 vols.; London, 1975), vol. ii, 143-165.

[23] Calogero, *Eleatismus*, 150.

[24] R. Le Poidevin, *Travels in Four Dimensions. The Enigmas of Space and Time* (Oxford, 2003), 156ff.

[25] Kirk, Raven and Schofield, *Presocratic Philosophers*, 273-274: 'The paradox in fact poses an incisive challenge to the attractive idea that motion must occur – if it occurs at all – in the present. It shows that it is hard to reconcile this idea with the equally attractive notion that in the present what moves cannot be traversing any distance'. See also Lear, 'Note', 91-104.

[26] E. Zeller, *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung dargestellt* [*Die Philosophie der Griechen*], Erster Teil: Allgemeine Einleitung, Vorsokratische Philosophie (Leipzig, 1869; 3rd edn.), 504, n. 2.

[27] This interpretation of the κατὰ-τὸ-ἴσον phrase was explicitly proposed by Renouvier (Ch. Renouvier, *Essais de critique générale* [*Essais*], vol. i (Paris, 1912), 43): '…Si toujours une chose est en repos quand elle est dans un espace égal à elle-même. C'est le concept même du repos… '. See also Lee, *Zeno*, 79: 'The whole point of the phrase κατὰ τὸ ἴσον must be that it is a definition of rest…'.

[28] Themistius, *Paraphrases Aristotelis*, I. Spengel (ed.), vol. i (Lipsiae, 1866), vi, 9: εἰ γὰρ ἠρεμεῖ, φησίν, ἅπαντα, ὅταν ᾖ κατὰ τὸ ἴσον αὑτῷ διάστημα, ἔστι δ' ἀεὶ τὸ φερόμενον κατὰ τὸ ἴσον ἑαυτῷ διάστημα, ἀκίνητον ἀνάγκη τὴν ὀϊστὸν εἶναι τὴν φερομένην. [*our underline*]

[29] Simplicius, *Commentaria in Aristotelis Physicorum libros quattuor posteriores*, Hermannus Diels (ed.) (Berolini, 1895), 1011, 19 (ad Arist. *Phys*. 239^{b}5): ὁ δὲ Ζήνωνος λόγος προλαβών, ὅτι πᾶν ὅταν ᾖ κατὰ τὸ ἴσον ἑαυτῷ ἢ κινεῖται ἢ ἠρεμεῖ …In 1011, 27-8 Simplicius literally reproduces the text in *Phys*. 239^{b}5-7 as reconsructed by Bekker: εἰπὼν δὲ ὅτι ἅπαν ἢ ἠρεμεῖ ἢ κινεῖται, ὅταν ᾖ κατὰ τὸ ἴσον, ἐπήγαγεν ἔστι δὲ ἀεὶ τὸ φερόμενον ἐν τῷ νῦν. [*our underline*]

[30] Philoponus, in 'Scholia in Aristotelem graeca', *Aristotelis opera* ex recensione I. Bekkeri, vol. iv (Berolini, 1961), 816. 30 (ad Arist. *Phys*. 239^{b}5): ἅπαν, φησίν, ἐν τῷ ἴσῳ ἑαυτῷ τόπῳ ὑπάρχον ἢ ἠρεμεῖ ἢ κινεῖται … [*our underline*]

[31] Renouvier, *Essais*, 43; Burnet, *EGP*, 319; W. D. Ross, *Aristotle's Physics* [*Physics*] (Oxford, 1960; 2nd edn.); Guthrie, *History of Greek Philosophy*, 93; Vlastos, 'Note', 184; Barnes, *Presocratic Philosophers*, 276; Kirk, Raven and Schofield, *Presocratic Philosophers*, 273; Lear, 'Note', 91; J. A. Faris, *The Paradoxes of Zeno* (Avebury, Aldershot, Brookfield, Singapore, Sidney, 1996), 37.

[32] According to Diels (H. Diels, *Die Fragmente der Vorsokratiker* [DK], W. Kranz (ed.) (3 vols., Dublin/Zürich, 1974; 6th revised edn.), DK 29 A 27), the omitted words are οὐδὲν δὲ κινεῖται (he is

followed by Lee, *Zeno*, 52f. and 78-82, and Chappell, 'Time', 198f.), according to Emminger (A. Emminger, *Die vorsokratischen Philosophen nach den Berichten des Aristoteles* (Würzburg, 1878), 43) οὐ κινεῖται δὲ, and according to Cornford, καὶ μὴ κινεῖται (Cornford, *Physics*, vol. ii, 180, n.1).

[33] This, however, does not hold for Lachelier's (J. Lachelier, 'Note sur les deux derniers arguments de Zénon d' Élée contre l' existence du mouvement', *Revue de Métaphysique et Morale* 18 (1910), 345-355 at 345) and Carteron's (Carteron, *Aristôte*, 60) suggestion that ἠρεμεῖ δ᾽ should be added here.

[34] The justification is not to be found in the occurrence of the words τῷ κατὰ τὸ ἴσον after ἐν τῷ νῦν in one of the manuscripts (codex Laurentianus 87. 7), since the sense gained through this addition is completely different from that which Zeller tries to obtain.

[35] Ross, *Physics*, 416: 'Zeno's argument is unsound. He says that if everything is at rest when it is over against that which is equal to itself, and that which is moving always fulfils this condition at a moment, the moving arrow must be at rest.'

[36] *Phys*. $204^{b}15$-20, $232^{a}13$-15, $242^{b}30$-34.

[37] O. Hamelin, 'Sur un point du troisième argument de Zénon contre le mouvement', *L'Année Philosophique* 17 (1907), 39-44.

[38] The last statement is also taken as a tacit premiss in the interpretation of V. Brochar, who believes this to be sufficient for the completion of the argument, taking as evident that there is no movement in an indivisible instant (see Brochard, 'Les arguments de Zénon', 7).

[39] Hamelin's reconstruction was recently supported by Shamsi. Shamsi tried to prove that 'there is just no reason to believe that (at *Physics* $239^{b}5$-7) Aristotle meant to summarize Zeno's argument of the arrow' (Shamsi, 'What Exactly Was Zeno's Argument of the Arrow?', 58) contrary to the fact that when citing one-by-one Zeno's four arguments on motion (*Phys*. $239^{b}10$-$240^{a}18$) Aristotle gives in lieu of the third argument merely a reference to $239^{b}5$-7 (τρίτος δὲ ὁ νῦν ῥηθείς, ὅτι ἡ ὀϊστὸς φερομένη ἕστηκεν).

[40] The version cited by Epiphanius (*Hae*r. III 11: τὸ κινούμενον ἤτοι ἐν ᾧ ἐστι τόπῳ κινεῖται ἤτοι ἐν ᾧ οὐκ ἔστι. καὶ οὔτε ἐν ᾧ ἐστι τόπῳ κινεῖται οὔτε ἐν ᾧ οὐκ ἔστιν) is longer and, in Fränkel's opinion, 'complies with the rules and conventions of post-Aristotelian syllogisms. The first version has a better claim to authenticity …' (H. Fränkel, 'Zeno of Elea's Attacks on Plurality' ['Zeno of Elea'], in R. E. Allen and D. J. Furley (eds.), *Studies in Presocratic Philosophy* (2 vols.; London, 1975), vol. ii, 102-142 at 106).

[41] Guthrie, *A History of Greek Philosophy*, 91.

[42] A. E. Taylor, *The Parmenides of Plato translated into English* (Oxford, 1934), 116.

[43] Fränkel, 'Zeno of Elea', 109.

[44] Calogero, *Eleatismus*, 151-152, n. 47.

[45] Τὸν περιφορητικὸν συνερωτᾷ λόγον εἰς τὸ μὴ κινεῖσθαί τι (see Fränkel, 'Zeno of Elea', 129, n. 20).

[46] See e.g. Barnes, *Presocratic Philosophers*, 276.

[47] Kirk, Raven and Schofield, *Presocratic Philosophers*, 273.

[48] Vlastos, 'Note', 187 and 196, n. 17.

[49] See Hippol. *Ref.* I 7, 2 = DK 13 A 7 and Simpl. *Phys*. 23, 33 (from Theophr. *Phys. Opin*. Fr. 1, D 475) = DK 22 A 5), Simpl. *Phys*. 23, 21; Simpl. *Phys*. 24, 26 (from Theophr. *Phys. Opin*. fr. 2. D. 476) = DK 13 A 5; Cic. *De nat. d*. I 10, 26 = DK 13 A 10; Aët. I 23, 27 = DK 22 A 6, etc.

[50] On the basis of Aristotle's remark in 239^{b}7-9 and 31-33 Tannery and his followers (see above, n. 21) supposed that, in the *Flying Arrow*, Zeno assumed that time is composed of indivisible quanta in order to refute the Pythagorean doctrine of time.

[51] Logicians define a *sound* argument as a valid argument all the premisses of which are true. The distinction between validity and soundness of reasoning is not marked in Aristotle's Greek.

[52] Συμβαίνει δὲ παρὰ τὸ λαμβάνειν τὸν χρόνον συγκεῖσθαι ἐκ τῶν νῦν.

[53] Diodorus expressed this view by stating that nothing can be said to be moving or in motion (κινεῖται μὲν οὐδὲ ἕν, in present tense), although something can be said to have moved, i.e., to have changed its position (κεκίνηται δέ, in perfect tense) (SE *M* 85-6).

[54] Vlastos, 'Note', 187-190 (cf. also notes 12, 13 and 14 on pp. 195-6).

[55] Herodotus III 62: Εἰ μέν νυν οἱ τεθνεῶτες ἀνεστᾶσι, προσδέκεό τοι καὶ Ἀστυάγεα τὸν Μήδον ἐπαναστήσεσθαι.

[56] In his arguments against plurality Zeno starts with the common belief and quite natural presupposition that there are many things, in order to show that such a presupposition leads to a contradiction, which means that multitude cannot exist. The protagonists of Zeno's arguments against motion are all first supposed to be in motion, and it turns out only later that their motion is impossible. One could object that in *The Dichotomy* the body is not even allowed to start moving. But, as some interpreters (e.g. Vlastos in

G. Vlastos, 'Zeno's Race Course', *Journal of the History of Philosophy* 4 (1966), 95-108 and Lee, *Zeno*, 67) have pointed out, Aristotle's text can be interpreted as referring to increasing geometric progression (and not only in Philoponus' way, where progression is descending). Zeno never starts with something confusing or odd, he prefers to surprise and shock after stating something commonly acceptable.

[57] Barnes, *Presocratic Philosophers*, 339, n. 14.

[58] Ross, *Physics*, 657.

[59] *Metaph*. 992^{b}18-24: ὅλως τε τὸ τῶν ὄντων ζητεῖν στοιχεῖα μὴ διελόντας, πολλαχῶς λεγομένων, ἀδύνατον ἐρεῖν … [*our underline*] This distinction between different meanings of a word, out of which one is primary, is applied by Aristotle already in *Eudemian Ethics* (1236^{a}18-23) where he considers the example 'medical'.

[60] In Arist. *Metaph*. 1004^{a}23-31 it is said: … ἐπειδὴ πολλαχὼς τὸ ἓν λέγεται, καὶ ταῦτα (viz. so-called species of unity: ταὐτὸ καὶ ὅμοιον and their opposites: ἕτερον καὶ ἀνόμοιον καὶ ἀνίσον, καὶ ὅσα ἄλλα λέγεται ἢ κατὰ ταῦτα ἢ κατὰ πλῆθος καὶ τὸ ἕν) μὲν λεχθήσεται …. Cf. *ibid*. ^{a}26.

[61] For this point it is irrelevant whether one assumes that there is a state of absolute rest (and consequently a state of absolute motion), as Aristotle does in *Phys*. 209^{a}32-33 where he introduces τόπος ὁ μὲν κοινός in which all bodies are situated (ἐν ᾧ ἅπαντα τὰ σώματά ἐστιν) or one assumes that the state of motion and the state of rest are always only relative.

[62] Owen was the first to show that Aristotle often approached difficult problems from the perspective of his theory of primary and secondary meaning (G. E. L. Owen, 'Logic and metaphysics in some earlier works of Aristotle', in J. Düring and G. E. L. Owen, *Aristotle and Plato in the Mid-Fourth Century* (Göteborg, 1960), 163-190).

[63] G. Cantor, 'Über unendliche, lineare Punktmannigfaltigkeiten', in *Gesammelte Abhandlungen*, edited by Ernst Zermelo (Hildesheim, 1962), 275 [*our translation*].

[64] *Ibid*. 194.

[65] B. Russell, *The Principles of Mathematics* (London, 1903), 469ff.

[66] See *ibid*. 441ff.

[67] For the comparison between the two theories see M. Arsenijević, 'Eine aristotelische Logik der Intervalle, die Cantorsche Logik der Punkte und die physikalischen und kinematischen Prädikate',

Philosophia naturalis, 29/2 (1992), 161-209 and M. Arsenijević 'Generalized concepts of syntactically and semantically trivial differences and instant-based and period-based time ontologies', *Journal of Applied Logic*, 1 (2003), 1-12.

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