To cite: Arsenijević, M. Determinism, Indeterminism And The Flow Of Time. *Erkenntnis* **56**, 123–150 (2002). https://doi.org/10.1023/A:1015646303123

DETERMINISM, INDETERMINISM AND THE FLOW OF TIME

Miloš Arsenijević

ABSTRACT. A set of axioms implicitly defining the standard, though not instant-based but interval-based, time topology is used as a basis to build a temporal modal logic of events. The whole apparatus contains neither past, present, and future operators nor indexicals, but only B-series relations and modal operators interpreted in the standard way. Determinism and indeterminism are then introduced into the logic of events via corresponding axioms. It is shown that, if "determinism" and "indeterminism" are understood in accordance with their "core" meaning, the way in which they are formally introduced here represents the only right way to do this, given that we restrict ourselves to one real world and make no use of the many real worlds assumption. But then the result is that the very truth conditions for sentences about indeterministic events imply the existence of tensed truths, in spite of the fact that these conditions are formulated (in the indeterministic axiom) in terms of tenseless language. The tenseless theory of time implies determinism, while indeterminism requires the flow of time assumption.

INTRODUCTION

The aim of this paper is to show that, if *determinism* and *indeterminism* are understood in accordance with the "core" meaning of the terms and defined in relation to an existing real world without taking into account a possible existence of more real worlds, then the tenseless theory of time implies determinism, while indeterminism requires the assumption that there is a flow of time.

This goal will be achieved by way of introducing deterministic and indeterministic axioms into a temporal logic of events that contains neither *indexicals* nor *past, present,* and *future* operators, but only *B-series* relations and *modal* operators interpreted in the *standard* way. This means, in effect, that the *flow of time* requirement will be a *consequence* of the *indeterministic* axiom formulated in *tenseless* language. Hence, the resulting preference for the *tensed* theory over the *tenseless* theory of time will have nothing to do with the reasons against the tenseless theory offered in Quentin Smith's *Language and Time*.[1] Moreover, since the logical apparatus as such does not make any use of *tensed* terms, this suggests that *determinists* should accept the *tenseless* view.

The main difference between the guiding idea here in sketching an indeterministic logic of events and the idea that guided Nuel Belnap in his "Branching Space-Time"[2] consists in the fact that Belnap's system was intended to show that the Special Theory of Relativity is compatible with indeterminism, while the system to be sketched here should establish only that the very branching of *possible worlds*, which as such has nothing to do with the branching of *time* but simply results from the assumption of *in-the-world-inherent possibilities*, requires also *the flow of time* assumption.

In order to avoid a possible relevance of relativistic effects, we may focus on a spatially localized segment of the world and treat this segment, for the sake of argument, as *the* real world, for, as we shall see, by taking into consideration just the "core" meaning of the term "determinism", we may allow the deterministic pattern to be of *any* kind, including its being the result of an influence from outside the world, be this influence the influence of God, or of someone or something else. Consequently, "indeterminism" will mean the absence of *any* deterministic pattern governing the “coupling” of (otherwise well specified) events and times at which they occur.

THE INTERVAL-BASED STANDARD TIME TOPOLOGY

Even if we use the word “event” as a general *terminus technicus* applicable freely and loosely in a variety of cases in everyday life and in science, it should be noted that at least predominantly if not exclusively “event” refers to something which lasts for a certain period of time. Not only wars and earthquakes, concerts and nuclear reactions, but also stabbing pains and photon transmissions from a light source to a very close backstop “occupy” some time intervals, however small. True, collisions understood as initial touchings of moving bodies, as well as, in general, beginnings and endings of those events which last for some time periods, can be said to be “events” that happen instantaneously, but we cannot speak of events in such a sense without speaking of events lasting for some time periods. So, it seems natural to sketch a system of *intervals* as time’s basic stuffs, and then enlarge it to a logic of events.

Though standardly defined with the use of an instant-based system, any time topology – discrete time, dense time, continuous time, branching or non-branching time, time with or without beginning and/or ending, etc. – can be also implicitly introduced *via* an appropriate interval-based system. This will become evident from the following time system in which variables will be directly interpretable as ranging over the basic set of intervals so that intervals will not be confined to a metalanguage as in propositional time logic.[3]

Let our system contain – besides logical constants $\neg$, $\Rightarrow$, $\wedge$, $\vee$ and $\Leftrightarrow$ – individual variables t_1, t_2, t_3,..., t_n,..., quantifiable by universal and existential quantifiers, as well as individual constants t_1, t_2, t_3,..., t_n,... (n = 1, 2, ...). Variables are supposed to range over time intervals, and constants to denote particular intervals. Let the system also contain the relation symbols =, $\prec$, $\wr$, $\cap$, and $\subset$, to be interpreted as identity, precedence, abutment, overlapping and inclusion relation respectively. The elementary *wffs* will be $t_1 = t_2$, $t_1 \prec t_2$, $t_1 \wr t_2$, $t_1 \cap t_2$ and $t_1 \subset t_2$ as well as any formulae obtainable by the substitution of t_1 and/or t_2 through some other variable(s) and/or constant(s). (For the way in which, given that time is linear, $\wr$, $\cap$, and $\subset$ are definable *via* = and $\prec$, see Appendix.)

Let us now introduce the axioms implicitly defining standard time topology. (In the Appendix, the *formal* reader can find the explanations of the meaning of the axioms, as well as the reasons why they are formulated in the way they are. The *informal* reader may, however, go directly to Section 3, by understanding the identity, precedence, abutment, overlapping and inclusion relations intuitively, and by simply taking into account that in what follows time is to be understood as linear, infinite, dense and continuous, independently of the fact that its basic stuffs are intervals and not instants.)

(A_1) $(\mathrm{t}_n)\neg(t_n \prec t_n)$

(A_2) $(t_k)(t_l)(t_m)(t_n)(t_k \prec t_m \wedge t_l \prec t_n \Rightarrow \mathrm{t}_k \prec t_n \vee t_l \prec t_m)$

(A_3) $(\mathrm{t}_m)(t_n)(t_m \prec t_n \Rightarrow t_m \wr t_n \vee (\exists t_l)(t_m \wr t_l \wedge t_l \wr t_n))$

(A_4) $(t_k)(t_l)(t_m)(t_n)(t_k \wr t_m \wedge t_k \wr t_n \wedge t_l \wr t_m \Rightarrow t_l \wr t_n)$

(A_5) $(t_k)(t_l)(t_m)(t_n)(t_k \wr t_l \wedge t_l \wr t_n \wedge t_k \wr t_m \wedge t_m \wr t_n \Rightarrow t_l = t_m)$

(A6) $(t_m)(\exists t_n)\ t_m \prec t_n$

(A7) $(t_m)(\exists t_n)\ t_n \prec t_m$

(A8) $(a_m)(\exists a_n)\ a_n \subset a_m$

(A9′) For any infinite sequence of intervals $t_1 t_2 \ldots t_i \ldots$ ordered by $\prec$, it holds that
$(\exists u)(\wedge_{i<\omega} t_i \prec u) \Rightarrow (\exists v)\ (\wedge_{i<\omega} t_i \prec v \wedge (w)(\wedge_{i<\omega} t_i \prec w \Rightarrow \neg(\exists x)(x \prec w \wedge \neg x \prec v))$

(A9″) For any infinite sequence of intervals $t_1 t_2 \ldots t_i \ldots$ ordered by $\succ$, it holds that
$(\exists u)(\wedge_{i<\omega} t_i \succ u) \Rightarrow (\exists v)\ (\wedge_{i<\omega} t_i \succ v \wedge (w)(\wedge_{i<\omega} t_i \succ w \Rightarrow \neg(\exists x)(x \succ w \wedge \neg x \succ v))$

A TEMPORAL MODAL LOGIC OF EVENTS

In view of the fact that our topic concerns not the question of the individuation of events in general but only the “coupling” of *events* and *time intervals*, various questions about *qualitative* and *spatial* aspects of the events’ individuation can be presupposed as settled in one way or other, in the case that they are not of relevance for what is our main concern. So, if the non-temporal aspects of individuation did not affect its temporal aspect, we could simply take that *e*, *e′*, *e″*,…are constants denoting qualitatively and spatially well-specified events of *any* kind. But there are certain important ambiguities that can arise and which require us to make some clarifications and stipulations in advance.

Let *e* denote *the raining in Graz*, and let the “coupling” of *e* with a time interval t_1 be denoted by $e(t_1)$. Does $e(t_1)$ *always* makes sense, independently of which particular interval is *de facto* denoted by t_1? In particular, *given* that $e(t_1)$ makes sense, does the coupling of *e* with *any* subinterval of t_1 also make sense?

It is a widespread belief (which I hold true) that time is *intrinsically metrically amorphous*.[4] In view of this *amorphousness* alone, we may answer the first question in the affirmative. But we want to speak of “couplings” (of events and time intervals) *from within* an already existing world. What then about *e* denoting the raining in Graz if t_1 is an interval which is a billionth of a second? For the sake of convenience, we could take that $e(t_1)$ still makes sense, but stipulate that the atomic sentence into which $e(t_1)$ is built, and which says that it is raining in Graz on interval t_1, is *false*. That is, if **A** is a sentence forming operator, the sentence $\mathbf{A}e(t_1)$ would be *false* but *not meaningless*. However, such a stipulation could, seemingly, lead us into great trouble.

Let us divide a day on which it was uninterruptedly raining in Graz into the set of abutting subintervals such that each is a billionth of a second long. Then, it should be said, according to the above stipulation that it was *not* raining in Graz on any of these intervals, in spite of the fact that they *make up* the day on which it was *uninterruptedly* raining. Now, in order to get rid of this unpleasant consequence, we may follow a suggestion Arthur Prior had given at the Conference in Oberwolfach shortly before he died.

Let us consider a similar case, in which a given surface is completely blue. By an appropriate partition of the surface, we may get the situation in which we should say that the surface *consists of* tiny parts which (being colorless themselves) are *not blue*. But isn’t it more natural to reverse the strategy and say that all these small parts *are* blue *because* they are parts of the blue surface? At least, this would make life easier. We may simply say that there are properties (and very many known properties, if not all, are such) that can be called *holistic*, and whose true ascription to a given area sometimes *depends* on whether they are truly ascribed to a larger area into which the given area is included.

Analogously, if on a given day it was raining in Graz uninterruptedly, we might say that it was raining *throughout* that day, also in the sense in which this means that it was raining on *any* of subintervals of the given day; and we shall henceforth take it that it is so.

But this is not the end of the story. There are certainly cases in which we want to say that it was raining *interruptedly*. This can hardly mean anything else but that there are subintervals on which it was not raining. How then to distinguish such cases from those in which we should say that it was raining on an interval because of the interval's inclusion into another interval on which it was raining? I see just one plausible answer to this question: It was raining *interruptedly* on a given interval if and only if there are both (i) subintervals of the given interval for which it can be said that it *was* raining *regardless* of whether they are included into intervals on which it was raining, as well as (ii) subintervals of the given interval for which it can be said that it was *not* raining *regardless* of whether they are included into intervals on which it was raining. Consequently, it was raining *uninterruptedly* on a given interval if and only if (i) is true and (ii) false, whereas it was not raining *at all* on a given interval if and only if (ii) is true and (i) false. Additionally, we stipulate that "it was raining" means "it was raining uninterruptedly", and that "it was raining in Graz" means "it was raining throughout Graz".

The above clarifications and stipulations have a major point: We avoid dealing explicitly with the questions concerning metrics and isotropy or anisotropy of time.[5] For our purposes, it is sufficient that *any* "coupling" of an event with a time interval *makes sense*, and that the evaluation of any atomic sentence is *consistently* and *unequivocally* determinable, where it is irrelevant whether for the evaluation of $\mathbf{A}e(t_1)$ we have to take into consideration the evaluation of some other sentence $\mathbf{A}e(t_2)$, where $t_1 \subset t_2$. What we *should* make *explicit* is only the differentiation between *elementary* events (such as *raining in Graz uninterruptedly*) and *complex* events (such as raining in Graz interruptedly), which we can do through the introduction of an axiom that I called elsewhere[6] *Prior's Axiom* (due to his suggestion in Oberwolfach). It should hold only for *elementary* events that *if* the sentence expressing an event's "coupling" with a time interval is true, *then* a sentence expressing its "coupling" with any subinterval of the given interval is also true. So, let us denote *elementary* events by bold letters: **e**, **e′**, **e″**; then *Prior's Axiom* reads as follows:

(A_{10}) $(t_n)(\mathbf{Ae}(t_n) \Rightarrow (t_m)(t_m \subset t_n \Rightarrow \mathbf{Ae}(t_m)))$.

Now, in order to deal with the question of determinism and indeterminism, we do *not* need to take *complex* events into consideration, if we strengthen our presupposition about the existing real world by presupposing that there is at least one elementary event occurring in it. It is sufficient that it is made certain that for any given value of t_n, the atomic sentence $\mathbf{Ae}(t_n)$ is either true or false, and that it is so for any substitution of **e′**, **e″**,…for **e** and/or any substitution of some other variable letter or constant for t_n.

Under above arrangements, the presupposition that there is at least one elementary event ceases to play any *metrics* related role, its purpose being only to tie our construction to a *real* world. At the same time, the *Leibnizian requirement* will be fulfilled (though in a weak sense). Namely, according to Leibniz, there would be no time if there were no real happenings.[7] But the *Leibnizian requirement* is fulfilled only in a weak sense, since we do not preclude that *within* the real world there are *empty* times, i.e., time intervals on which *nothing* happens.[8] The *strict* Leibnizians would not use *standard* time topology in such a case, but, in view of our purposes, we should rule out *time reductionism* in order not to preclude the possibility of speaking about times of *not-yet-actualized* events. What we need for our argument is just a *possible worlds* semantics.

As promised, the possible worlds semantics will be *standard*, i.e., we shall take $\Box_w$**Ae**(t_n) and $\Diamond_w$**Ae**(t_n) to mean, respectively, that **Ae**(t_n) is true in *all the worlds accessible from within the world w*, and that **Ae**(t_n) is true in *some worlds accessible from within the world w*. The accessibility relation is to be taken as a primitive. Whether it will be reflexive or nonreflexive, symmetric or non-symmetric, transitive or non- transitive, is to depend on a particular modal logic system within which it is introduced according to a particular intended context of its interpretation. At the moment, we leave it open whether the system we are looking for is to be S5, in which case the accessibility relation would be an equivalence relation (for, according to the interpretation of S5, each possible world is accessible from within any of the possible worlds) or the system is to be a weaker one. Anyway, I take that the "core" meaning of "from within *w*, it is necessary that ..." and "from within *w*, it is possible that ..." is that the sentence following the first and second phrase *is true in all the worlds accessible from* w, and *true in some possible worlds accessible from* w, respectively, while the more specific meanings will emerge only later, as a *consequence* of introducing deterministic and indeterministic axioms into our logic of events.

Of course, not only atomic sentences but also any complex formula will be allowed to be prefixed by box- or diamond-operator.

DETERMINISTIC VERSUS INDETERMINISTIC LOGIC OF EVENTS

Now we turn to the difference between deterministic and indeterministic universes to be expressed axiomatically in our logic of events. As for determinism, we shall take as irrelevant which specific kind of determination requires us to treat events deterministically. This may be due to the existence of a causal chain (or causal chains), or due to 1 to 1 correspondence between events and time intervals pre-established by God, or for any other reason whatsoever. What is sufficient is that any sentence stating that an event occurs on a time interval, though *logically contingent*, is either *necessarily true or necessarily false*. So, in order to formulate the deterministic axiom, we have just to sort out a set of possible worlds and specify the accessibility relation, since "necessarily true" means "true in all the possible worlds accessible from within a given world".

The *set of possible worlds* is to be the set of all the worlds in which, for any elementary event, say **e**, and any time interval, say t_n, it holds that either **Ae**(t_n) or ¬**Ae**(t_n) is true. Now, since we want to define determinism under the *one real world* assumption, the accessibility we are interested in is to be defined as the *accessibility from within a possible world that is a segment of the real world on a time interval*, where, at the same time, *an actual world* is to be identified with *a time segment of the real world*. As for the possible worlds that are to be understood as accessible (from within a real world segment), they will be taken, in a standard way, to be all those worlds that are *actualizable* starting from within a given time segment of the real world. (Notice that all actual worlds are to be understood as *ipso facto* actualizable, whereas even if it *turns out* that all actualizable worlds are segments of the real world, it is not so *per definitionem*!).

As a *link* with the real world, we may use the fact that the weak Leibnizian requirement is supposed to be fulfilled. Let us suppose that the requirement is fulfilled by the fact that the real world is a world in which an event **e** occurs on time interval t_2. Then, for the sake of convenience, we shall simply speak of the possible worlds accessible from t_2, meaning *the possible worlds accessible from the real world segment on* t_2. Generalizing, for any *n*, by speaking of *the possible worlds accessible from* t_n, we shall always mean *the possible worlds accessible from the real world segment on* t_n. Consequently, by using modal operators, we shall simply drop out subscripts specifying worlds from within which we want to speak about necessity or possibility, and, instead of that, prefix the given operator by $[t_1]$ or $[t_2]$ or … or $[t_n]$ or .... So, for instance, $[t_1]\Box\ldots$ is to mean that the formula following the box-operator is true in all the possible worlds accessible from the real world segment on t_1. Moreover, if a modal operator lies within the scope of a quantifier, we may omit even a particular time specification altogether, for we can stipulate that $(t_n)\ldots\Box\ldots$ and $(\exists t_n)\ldots\Box\ldots$ mean that the formula following the box-operator is true in *all* the possible worlds accessible from within *any* (*some*) real world segment, whereas $(t_n)\ldots\Diamond\ldots$ and $(\exists t_n)\ldots\Diamond\ldots$ mean that the formula following the diamond-operator is true in some possible worlds accessible from *any* (*some*) real world segment.

Now, what should it mean that, viewing from within the real world segment on an interval, say, t_2, an event, say **e**, is necessarily coupled with an interval, say t_n? This can mean nothing else but that $[t_2]\Box\mathbf{Ae}(t_n)$ is true. Consequently, it means that a possible world in which **e** doesn't occur on t_n belongs to inaccessible possible worlds. For if such a world were accessible, $\mathbf{Ae}(t_n)$ would not be true in all the possible worlds accessible from within the real world segment on t_2 (as it should be according to the reading of $[t_2]\Box\mathbf{Ae}(t_n)$). Analogously, if $\mathbf{Ae}(t_n)$ were false, $[t_2]\Box\neg\mathbf{Ae}(t_n)$ would be true. So, we may safely infer

$$[t_2](\Box\mathbf{AE}(t_n) \vee \Box\neg\mathbf{AE}(t_n)).$$

Generalizing, and taking that the Leibnizian requirement is fulfilled in relation to an event *e* occuring on t_2, we can formulate the deterministic axiom, $\mathbf{A}_{\mathbf{11(d)}}$, as follows:

($A_{11(d)}$) Given that $\mathbf{Ae}(t_2)$ is true in the real world,

$$\mathbf{Ae}(t_2) \Rightarrow (t_n)(\Box\mathbf{AE}(t_n) \vee \Box\neg\mathbf{AE}(t_n)),$$

where **E** is a schematic letter substitutable through **e**, **e**′, **e**″,….

This deterministic axiom may sound trivial and redundant. However, it is not trivial, and it is redundant just in the sense in which it should be. It is not trivial because it cannot be obtained by modalizing logical truths. Accepting the Principle of Bivalence and the Principle of Excluded Middle, we can obtain at best $(t_n)\Box(\mathbf{AE}(t_n) \vee \neg\mathbf{AE}(t_n))$, which *does not* imply $(t_n)(\Box\mathbf{AE}(t_n) \vee \Box\neg\mathbf{AE}(t_n))$. The axiom can be said to be redundant in the sense that, though allegedly being about *necessity*, it *de facto* makes all truths *modally indiscriminative* (given that they are viewed from within the real world), since it implies that *every* sentence about the occurrence or non-occurrence of an event on a time interval is either true *simpliciter* or false *simpliciter*, always allowing *necessitation* either of the affirmation or of the negation of any sentence. This means, in effect, the *collapse* of the difference between modalities. However, such a collapse is precisely what complete determinism *means*, or at least *implies*: Within a completely deterministic world there are *no real* possibilities,[9] i.e., *in-the-world-inherent possibilities*,[10] since everything that *is*, is *necessarily* such as it is.[11] The sole possibilities that still remain are the *purely logical possibilities*, which survive only due to the fact that there are truths that, though *necessary*, are not *logically* necessary. The negations of truths that are *necessary*, but not *logically* necessary, could have been true only in *inaccessible* possible worlds, which is the fact that still does not change the modal status of all truths, since "being necessary true" means "being true in all *accessible* possible worlds".

The same point can be expressed through the fact that the *accessibility relation* has *turned out* to be an *equivalence relation definable on the set of all real world time segments*, meaning that there is *no actualizable world* that is not a *real world segment*[12] (where it is important to notice that this is a consequence of the *way* in which *the deterministic axiom* had to be introduced according to its *intended meaning*, and not a consequence of the *definition* of the accessibility relation, according to which the accessible worlds are simply those worlds that are actualizable).

Now we see that in a *non-completely* deterministic world there must be sentences, at least one, such that, viewed from within some real world time segment, the disjunction, which has the necessitation of such a sentence and the necessitation of its negation as disjuncts, does not hold. Let $\mathbf{Ae}'(t_3)$ be such a sentence. What should be said of it? By simply erasing the two box-operators in $\Box\mathbf{Ae}'(t_3) \vee \Box\neg\mathbf{Ae}'(t_3)$, we obtain nothing else but an instance of the Principle of Excluded Middle. Definitely, we want to say more. Putting diamonds instead of box-operators would still be too weak. What we actually want to have is that the two possibilities, that of $\mathbf{Ae}'(t_3)$ and that of $\neg\mathbf{Ae}'(t_3)$, are so related that $\mathbf{Ae}'(t_3)$ is *just possible*, and *not more* than that, *because* $\neg\mathbf{Ae}'(t_3)$ is *also possible*, and *vice versa*. So, we have to change our disjunction into conjunction $\Diamond\mathbf{Ae}'(t_3) \wedge \Diamond\neg\mathbf{Ae}'(t_3)$.

If we are now to look for an interpretation under which the conjunction is true, it is clear that we need *two* sets of worlds that are of the *same modal status*, in the sense that they are *both accessible*, so that the truth of $\mathbf{Ae}'(t_3)$ in a world of one of the two sets is what makes $\Diamond\mathbf{Ae}'(t_3)$ true, while the truth of $\neg\mathbf{Ae}'(t_3)$ in a world of the other set is what makes $\Diamond\neg\mathbf{Ae}'(t_3)$ true. The reason why the two sets of worlds must both be accessible lies in the fact that the two conjuncts ought to be true in the *same sense*. In other words, since the sense in which they are true consists in the fact that each of them is true in just one of the two sets of possible worlds in which the other conjunct is false, both sets must be sets of *accessible possible* worlds. At the same time, however, we mustn't allow for the possibility that some two worlds, one, in which $\mathbf{Ae}'(t_3)$ is true, the other, in which $\neg\mathbf{Ae}'(t_3)$ is true, *both* become actual, unless we endorse Lewis' modal realism[13] (which we don't, due to the one real world presupposition). So, the fact that the two sets of possible worlds are both to be accessible requires that we take *being actualizable* as not implying *being a segment of a real world*: The set of all real world time segments is a *proper subset* of the set of actalizable worlds. The *accessibility relation* ceases to be an equivalence relation definable on the set of all real world time segments just due to the fact that it is *not symmetric* any more. Namely, even if we stipulate (as we do) that by speaking about accessibility we are supposed to start from within a real world time segment, an accessible world is, though actualizable *per definitionem*, not necessarily actual, i.e., not necessarily a real world time segment. (Note that we speak of two *sets* – and not just of two *worlds* – in which one of the two contradictory sentences is true and the other false because various events ($\mathbf{e}$, $\mathbf{e}'$, $\mathbf{e}''$,…) can, but don't have to, occur on the *same* time interval in *one and the same* world, so that the world in which the sentence $\mathbf{Ae}(t_3)$ is true can be the world in which $\mathbf{Ae}'(t_3)$ and/or $\mathbf{Ae}''(t_3)$ and/or … are true as well as the world in which $\neg\mathbf{Ae}'(t_3)$ and/or $\neg\mathbf{Ae}''(t_3)$ and/or … are true. The same hold, *mutatis mutandis*, for $\neg\mathbf{Ae}(t_3)$.)

The given interpretation enables us to explain how both $\Diamond\mathbf{Ae}'(t_3)$ and $\Diamond\neg\mathbf{Ae}'(t_3)$ can be true, but what about $\mathbf{Ae}'(t_3)$ and $\neg\mathbf{Ae}'(t_3)$? Neither of the two may be said to be true *simpliciter* or false *simpliciter*. If we do not want to follow Lukasievicz in rejecting the Principle of Bivalence[14] (as I don't), we may simply say that ascribing truth and falsity to $\mathbf{Ae}'(t_3)$ and $\neg\mathbf{Ae}'(t_3)$ needs a *qualification*, the reason for that being the fact that $\mathbf{Ae}'(t_3)$ and $\neg\mathbf{Ae}'(t_3)$ both relate to *two* different sets of worlds of the *same* modal status: $\mathbf{Ae}'(t_3)$ is true in one of the sets of worlds, in which $\neg\mathbf{Ae}'(t_3)$ is false, while $\neg\mathbf{Ae}'(t_3)$ is true in the other set of worlds, in which

$\mathbf{Ae}'(t_3)$ is false. *Nothing else* could be said and should be said about the truth and falsity of $\mathbf{Ae}'(t_3)$ and $\neg\mathbf{Ae}'(t_3)$ *when* both $\Diamond\mathbf{Ae}'(t_3)$ and $\Diamond\neg\mathbf{Ae}'(t_3)$ are true, and *vice versa*: *When* nothing more could be said and should be said about the truth and falsity of $\mathbf{Ae}'(t_3)$ and $\neg\mathbf{Ae}'(t_3)$ but that they are true (false) in *differently qualified senses* (i.e., in respect of two different sets of worlds), then $\Diamond\mathbf{Ae}'(t_3)$ and $\Diamond\neg\mathbf{Ae}'(t_3)$ are *both true*. But *when* is this so, i.e., *which real* situation is such that it offers two sets of possible worlds having the same modal status so that two contradictory sentences could both be true as a consequence of their different qualifications?

The answer to this question is readily available: If $\mathbf{e}'(t_3)$ is *indeterministic*, as it is assumed to be, then $\mathbf{Ae}'(t_3)$ and $\neg\mathbf{Ae}'(t_3)$ are neither true nor false *simpliciter*, while $\Diamond\mathbf{Ae}'(t_3)$ and $\Diamond\neg\mathbf{Ae}'(t_3)$ are both true, at least at *all times* (on all intervals) that *precede* t_3, since on these intervals possible worlds in which $\mathbf{Ae}'(t_3)$ and $\neg\mathbf{Ae}'(t_3)$ are true have the *same modal status*, *neither* of them being as yet actualized. It is equally clear that $\neg\mathbf{Ae}'(t_3)$ and $\Diamond\neg\mathbf{Ae}'(t_3)$ cannot be true at any time (on any time interval) that is *later* than t_3, since one particular possible world (in which either $\mathbf{Ae}'(t_3)$ or $\neg\mathbf{Ae}'(t_3)$ is true) has a *privileged* modal position, because (*pace* David Lewis) the *actualized* world (in which, say, $\mathbf{Ae}'(t_3)$ is true) differs from the *unactualized* one (in which $\neg\mathbf{Ae}'(t_3)$ *would have been* true) just in view of their *modal status*: The former, as *actualized*, is a segment of the *real world*, and the latter (therefore!) *inaccessible* (i.e., *unactualizable*).

The final, a bit more complicated case is one in which the truth or falsity is to be ascribed to $\Diamond\mathbf{Ae}'(t_3) \wedge \Diamond\neg\mathbf{Ae}'(t_3)$ on intervals that *overlap* with t_3 or in which it is the case that either these intervals are *included* in t_3 or t_3 is *included* in them. Due to (A_{10}), according to which "being true on an interval" implies "being true on any of its subintervals", we have to accept that $\Diamond\mathbf{Ae}'(t_3) \wedge \Diamond\neg\mathbf{Ae}'(t_3)$ is *false* on any interval t_k in which t_3 is included or which is included in t_3 in such a way that no subinterval of t_3 is later than t_k, as well as on any interval t_m overlapping with t_3 in such a way that there is a subinterval of t_m that is later than t_3. The only remaining case, in which $\Diamond\mathbf{Ae}'(t_3) \wedge \Diamond\neg\mathbf{Ae}'(t_3)$ *can be true*, is the case of an interval t_n that overlaps with t_3 in such a way that there is a subinterval of t_3 that is later than t_n, for in such a case there is a "part" of $\mathbf{e}(t_3)$ whose occurrence is as possible as its non-occurrence. However, in such a case the truth of $\Diamond\mathbf{Ae}'(t_3) \wedge \Diamond\neg\mathbf{Ae}'(t_3)$ depends *also* on whether the "rest" of $\mathbf{e}'(t_3)$ is actualized or not: If it is *not*, then $\neg\,\mathbf{Ae}'(t_3) \wedge \Diamond\neg\mathbf{Ae}'(t_3)$ is *false*.

We are now ready to formulate the *indeterministic axiom*, ($A_{11(\mathrm{in})}$), by generalizing from what has been said regarding $\mathbf{e}'(t_3)$ to all events denoted by bold letters, as if there is no elementary event actualized according to a deterministic pattern. In order to fulfil the Leibnizian requirement, we shall again suppose that for some $\mathbf{e}$ and some t_5, $\mathbf{Ae}(t_5)$ is true *simliciter* (which only means that we are speaking from within a world in which $\mathbf{e}(t_5)$ is supposed to be actualized, and not that $\mathbf{e}(t_5)$ is actualized according to a deterministic pattern). Hence,

($A_{11(\mathrm{in})}$) Given that $\mathbf{Ae}(t_5)$ is true *simpliciter*,

$$\mathbf{Ae}(t_5) \Rightarrow ((t_n)(\Diamond\mathbf{AE}(t_m) \wedge \Diamond\neg\mathbf{AE}(t_m) \Leftrightarrow$$

$$\Leftrightarrow t_n \prec t_m \vee (\exists t_k)(t_n \{ t_k \wedge t_k \subset t_m \wedge (\mathbf{AE}(t_k) \Rightarrow \mathbf{AE}(t_m))))).$$

(For the explanation of the last two strings, notice that the possibility that an event occurs and that it doesn't occur on t_m – $\Diamond\mathbf{AE}(t_m) \wedge \Diamond\neg\mathbf{AE}(t_m)$ – is unquestionable on an interval t_n if t_n precedes t_m – $t_n \prec t_m$ –, but still exists also in the case in which t_n overlaps or is included in t_m, given that, at the same time, the possibility that $\mathbf{E}$ occurs on t_m is still not precluded, in the sense that the fact that the occurrence of $\mathbf{E}$ on an interval t_k, which abuts t_n and is included in t_m – $t_n \{ t_k \wedge t_k \subset t_m$ –, would make $\mathbf{AE}(t_m)$ true – $\mathbf{AE}(t_k) \Rightarrow \mathbf{AE}(t_m)$.)

Before investigating whether the choice between the deterministic and indeterministic axiom has to do with the question about the existence of the flow of time, I want to go once again, in a more detailed way, through all the relevant presuppositions that led us to formulate the two axioms in the way in which we did, in order to show why I believe that this was not just *a* right way but the *only* right way of doing that.

First of all, standard, though not instant-based but period-based time topology, as well as the logic of events built up on it, do not presuppose by any means the so-called *tensed* theory of time. We did not make any use of the concepts of *pastness*, *presentness*, and *futurity*, either explicitly, by using *past*, *present*, and *future* operators, or implicitly, by somehow building the indexical *now* into the truth conditions for atomic sentences. The logic of events was formulated in terms of the so-called *B*-series. This means that *if* we later reach the conclusion that there is a flow of time, this must be a consequence of the deterministic and/or indeterministic axiom(s).

Concern that our use of *modal* operators could be responsible for the far-reaching consequence that there is a flow of time required us to be quite clear and explicit about the use of modal concepts. Again, expressions “it is possible that ...” and “it is necessary that ...” were interpreted in the *standard way*, and I took this interpretation to represent the “core” meaning of “being possible” and “being necessary” that can be expanded on only by sketching *particular* modal logic systems or by *combining* the *pure modal* logic with some *other* logic, say, as in our case, with a temporal logic of events. So, if it turns out that there is a flow of time, it will not be so because of a non-standard use of the diamond- and box-operators, but because of the way they function *when combined* with sentences containing temporal qualifications.[15]

As for the distinction between “real” and “possible”, Lewis’ *modal realism* was not presupposed because of its extreme ontological commitments according to which all possible worlds are equally real, while the world that we *normally* consider real is just one of those worlds singled out by the *indexical* “our”. I adopted instead the *ordinary* view, and so we had to distinguish determinism and indeterminism for the case in which there is just *one* real world. This does *not* mean that the *possibility* that there are more real worlds was *precluded*. It was simply *not taken into account*.

It was mentioned above that the logic of events as it had been formulated, and within which determinism and indeterminism were later defined, did not presuppose the tensed theory of time. But this does *not* mean that the truth of the *tenseless* theory of time was presupposed either. It was *left open* to be seen whether the definitions of determinism and indeterminism, implicitly given through deterministic and indeterministic axioms, demand the denial of the *tenseless theory* or not.

All in all, as we began our discussion of how to understand determinism and indeterminism and how to introduce them axiomatically into our logic of events, the set of relevant presuppositions was reduced as much as possible to those presuppositions which seem comparatively weak and uncontested (and something must always be presupposed anyway). The logic of events was formulated in terms of a *B*-series, but the truth of the tenseless theory of time was not presupposed; the diamond- and boxoperators were understood in accordance with the “core” meaning of their standard interpretation, but its further qualifications were left open; the possibility that there are more real worlds was not precluded, but it was not presupposed either, so that the task of defining determinism and indeterminism was reduced to one real world whose existence is uncontested; to fulfil the weak Leibnizian requirement, the truth of a sentence about an event occurring in the one real world was presupposed, though we had to rule out the trivial case where the reductionist theory of time is defined as a set of events, which has the result that every event in the set belongs necessarily to the time and the time necessarily exists with the events, for in such a trivial case we would lose a mechanism for speaking about times of not-yet-actualized events; non-branching time topology was vindicated, in view of possibly relativistic effects, through the restriction of the domain of events to those occurring within a spatially localized world segment. Hence, given this set of standard and exceedingly weak presuppositions, the crucial move – which one might expect could sort out the competing doctrines – consisted in introducing determinism and indeterminism axiomatically.

What belongs to the "core" meaning of "determinism", and which could hardly be falsified by a historical analysis of the term, is the implication that *whatever* the reason for the "coupling" of an event and a time interval may be, the two, *if* "coupled", are "coupled" *necessarily*, not only because the event would not be *that very* event if it didn't occupy the given time interval, but also because the time interval *could not exist* without being occupied by the given event. This can be understood in no other way but as the equivalent of saying that the sentence expressing some "coupling" is, if true, true *necessarily*, while its negation is then not only false but *impossible*, i.e., *false necessarily*. In accordance with the standard possible worlds semantics (which *has been* presupposed), the last fact means that the set of all contradictory sentences about the "couplings" of events and intervals can be divided into two exclusive sets, one containing all and only sentences that are *true* in the *real* world (and, per our goal, determinism *had to be* defined by presupposing just one real world), the other containing all and only sentences that are *false* in the *real* world while *true* in no *accessible* possible world (for if there were such a world, the sentences would not be *necessarily* false). So, it turns out that according to the concept of determinism defined in relation to just one real world, there is always just *one accessible possible* world, and it is a real world segment, while all other *possible* worlds (possible because not all truths are truths of logic) are *inaccessible*. Since on *all* the intervals *each* true sentence is necessary as a result of being true in only *one* accessible possible world – the one that is a real world segment – different modalities became *indistinguishable*. Determinism implies (under our presuppositions) the *collapse* of different modalities into the just one: *reality*, and so I do not see *any* other way to express determinism under the above presuppositions but through our axiom ($A_{11(d)}$), which – saying that (given that, for some **e** and some t_2, $\mathbf{Ae}(t_2)$ is true) it holds that $\mathbf{Ae}(t_2) \Rightarrow (t_n)(\Box\mathbf{AE}(t_n) \vee \Box\neg\mathbf{AE}(t_n))$ – entails the *coextensiveness* of the sets of true and necessarily true sentences.

It is clear that the adoption of ($A_{11(d)}$) does not require the temporal relativization of truths, and those who accept any tenseless analysis of tensed sentences – be it *via* token-reflexive truth conditions of tensed sentences,[16] *via* their utterance dates,[17] *via* a co-reporting nature of tensed and tenseless sentences,[18] or *via* a contextualization of tensed sentence types[19] – have no reason to worry about ($A_{11(d)}$): *Determinism* and the *tenseless theory of time are compatible*. However, might the connection between them be even stronger?

In contrast to deterministic events, an indeterministic event is not necessarily "coupled" with a time interval, meaning that the sentence expressing this "coupling" and the negation of this sentence are *both possibly true*. This means further, according to the standard possible worlds semantics, that there are two exclusive sets of accessible possible worlds, one in which the affirmation of the sentence is true (and its negation false), and the other in which the negation of the sentence is true (and its affirmation false). Now, a difficulty arises. If *any* world from one of the two sets of worlds were a real world segment, then, given that we want to have an indeterministic axiom that would hold even if there were just one real world, *all other* worlds would have to be *merely* and *not really* possible, i.e., they would have to be *inaccessible*. This means that, if it *ought not* to be so, *no* accessible possible worlds should be a *real world segment*. Fortunately, the difficulty can be overcome due to the fact that our framework is not just a *modal* logic system, but a *temporal* modal logic system. The claim that *no* accessible world is a real world segment is not to be understood, in the given case, as something true *simpliciter*, but as meaning that it is so only under certain *temporal qualifications*: No world from the two sets of worlds is to be said to be a real world segment (i) on any time interval *preceding* the indicated time of the event's (possible) occurrence, and (ii) on any intervals overlapping with, or being included in, the indicated time of the event's occurrence if the happenings in the real world before the end of those intervals are such that the full occurrence of the event is neither precluded nor guaranteed. So again, I do not see *any* other way to formulate the indeterministic axiom under the set of presuppositions cited above but to do it as it was done in ($A_{11(in)}$), which – saying that (given that, for some **e** and some t_5, $\mathbf{Ae}(t_5)$ is true *simpliciter*) it holds that

$$\mathbf{Ae}(t_5) \Rightarrow ((t_n)(\Diamond\mathbf{AE}(t_m) \wedge \Diamond\neg\mathbf{AE}(t_m) \Leftrightarrow$$

$$\Leftrightarrow t_n \prec t_m \vee (\exists t_k)(t_n \rfloor t_k \wedge t_k \subset t_m \wedge (\mathbf{AE}(t_k) \Rightarrow \mathbf{AE}(t_m))))).$$

– entails the *non-coextensiveness* of the sets of true and possibly true sentences (concerning the events that happen after t_5).

In contrast to ($A_{11(d)}$), the indeterministic axiom ($A_{11(in)}$) *implies* the *temporal relativization* of some truths and falsehoods, for the right side of the equivalence in ($A_{11(in)}$) specifies exactly on which time intervals the left side of the equivalence is true and on which time intervals it is false. For instance, if **E** is substituted for by **e** that denotes *the raining in Graz*, ($A_{11(in)}$) says that $\Diamond\mathbf{AE}(t_m) \wedge \Diamond\neg\mathbf{AE}(t_m)$ is true (otherwise being false) on all intervals that precede t_m, as well as on all intervals which overlap with, or are included in, t_m given that before they elapse it was raining in Graz on the (whole proper) subinterval of t_m whose elapsing coincides with their elapsing. If the constant t_{29}, denoting, say, 29 March 2000, were substituted for t_m, then $\Diamond\mathbf{AE}(t_{29}) \wedge \Diamond\neg\mathbf{AE}(t_{29})$ would be true before 29 March 2000, as well as the entire day of 29 March 2000 before midnight, given that during that time it was raining in Graz. In all other cases related to 29 March 2000, as well as after 29 March 2000, $\Diamond\mathbf{AE}(t_{29}) \wedge \Diamond\neg\mathbf{AE}(t_{29})$ would be false.

So, by letting t_n in ($A_{11(in)}$) range over the set of all intervals, we get that the equivalence is *always* true, but only because the formulae flanking the equivalence sign are, for *any* pair of values of t_n and t_m, either *both true* or *both false*, and *not* because they are always true. It is important to notice that temporal relativization of truths and falsehoods holds for *all* atomic sentences prefixed by diamond-operator, including the sentence $\mathbf{Ae}(t_5)$ itself that occurs in the antecedent of the axiom. For, by substituting $\mathbf{Ae}(t_5)$ for $\mathbf{AE}(t_m)$ in ($A_{11(in)}$), we see that, though $\mathbf{Ae}(t_5)$ is supposed to be true *simpliciter*, it is so only for time intervals later than t_5. In particular, on intervals earlier than t_5, $\Diamond\mathbf{Ae}(t_5) \wedge \Diamond\neg\mathbf{Ae}(t_5)$ was true. This means that ($A_{11(in)}$) is about the indeterministic events occurring, or not occurring, in a world in which $\mathbf{Ae}(t_5)$ is *de facto* actualized, though it *could have been* otherwise according to ($A_{11(in)}$) itself.

Now, the fact that two contradictory atomic sentences are both possibly true on some time intervals (i.e., both true and false under different qualifications), while on some (later) time intervals one of them is to be true *simpliciter* and the other false *simpliciter*), requires that time *flows*, and moreover, that it *flows from earlier towards later* intervals (in direction that will be henceforth called the *positive* direction). For it is only such a flow of time that enables us to say *without contradiction* both (i) that the *information content* of *any* of the two contradictory sentences is *actualizable* in an accessible possible world, and (ii) that the *information content* of one of the two sentences is *actualizable* in *no* accessible possible world. *Both* cannot be true at the same time, but neither *should* both be true at the same time, but the former true at one time and the latter true at *a later time*.

It is important to see why the assumption that time flows in the *positive* direction *does* the job, while the assumption that time flows in the opposite (*negative*) direction *does not*. The assumption that time flows in *positive* direction works because by flowing in such a way time always *turns* an accessible possible worlds (in which one of a pair of contradictory sentences is true and the other false) into an *actual* world, making by this very transformation other possible worlds (in which the sentence true in now actual world is false) *inaccessible*, and this is just what we need to have in view of ($A_{11(in)}$). If time were flowing in the *negative* direction, time would turn an *actual* world into a non-actual (because otherwise the possible worlds from the other set of worlds could not become *accessible* possible worlds). This loss of actuality (that the once actual world suffered) need not be unacceptable *as such*. This is just what the so-called *presentism* implies.[20] However, in *our* case, it does have an unacceptable consequence. Since *other*, once *non-actual* worlds had to become *accessible* possible worlds, they would have to be *actualizable*, and for this to be the case, time would have to be permitted to flow in the positive direction. And by allowing time to flow in the *positive* direction *after* it had flown in the negative direction, we could obtain that an event occurred and did not occur on the same time interval, which our *one-real-world-presupposition* precludes: If the occurrence of an event on a time interval belongs to the real world, its non-occurrence cannot belong to a real world.

The fact that the flow of time serves well, under our presuppositions, only if time flows in the *positive* direction, means, *inter alia*, that *the* (presupposed) real world is to be understood as a world that *comes-into-being* through the flow of time, where the *not-yet-actualized* parts of it are necessarily *later* than those which are *actualized*. This is of great importance in view of the fact that our *topological* axioms together with *Prior's Axiom* (A_1)–(A_{10}) allow the *earlier-than* relation to be *substituted for* by the *laterthan* relation with no consequences concerning the axioms' satisfiability in a relational structure: In all relational structures in which the axioms are satisfied by containing the *earlier-than* relation, they are also satisfied by containing the *later-than* relation instead, and *vice versa*. Adding ($A_{11(d)}$) to (A_1)–(A_{10}) does *not* change this situation, but the addition of ($A_{11(in)}$) *does*. This means that it is only the introduction of the *indeterministic* axiom that which, under our presuppositions, *requires*, i.e., *implicitly defines* (in Hilbertian sense) the *flow* and *arrow* of time.

CONCLUSION

We saw above that *determinism* and the *tenseless* theory of time are *compatible*. Now, the relation between the two may be strengthened in view of the result concerning the relation between *indeterminism* and *temporal relativization* of some truths and falsehoods. By *precluding indeterminism*, the *tenseless* theory *implies determinism*. This cannot be said, of course, without qualification given by our presuppositions, but it should be repeated that these presuppositions are standard and minimal, just as the definitions of "determinism" and "indeterminism" we have used concern only the "core" meaning of the two terms which is hardly contestable from the historical point of view. In particular, determinism could be avoided within a tenseless theory if it presupposed Lewis' *plurality of real worlds*,[21] but I do not know any *detenser* who *explicitly* endorses Lewis' view, and, in addition, the *plurality of real worlds* assumption is certainly much, much stronger than our *one real world* assumption. Determinism could also be avoided by stressing *just* some *specific* meaning and *not mentioning* the "core" meaning of the term. For instance, one could stress, as Hugh Mellor does, the *causal* *chain* connecting events,[22] and claim that its non-existence implies *indeterminism*. But this move *alone* would not do the job without also rejecting the "core" meaning according to which there is a *possibility* that particular events and particular time intervals be "coupled" *as well as* a possibility that they not be "coupled", since on the *tenseless* view they are either "coupled" *once forever* or "noncoupled" *once forever, regardless* of whether they are deterministic or indeterministic in some deviant sense.[23]

Another important thing to be stressed is the fact that our account of indeterminism, though implying the truth of the *tensed* theory of time, does not make any use of *indexicals*, for the account is only about *conditions* under which indeterminism is true and *not* about any *particular* tensed truth. The equivalence in ($A_{11(in)}$), which states these conditions, is true *tenselessly*, though the conditions are about truths, which are *tensed*. It would be simply *wrong* to infer that there are not tensed truths on the basis of the sole fact that it is formulable tenselessly when these truths are true, just as the fact that there are tensed truths does not imply that the conditions under which it is so cannot be formulated in a tenseless manner. True, the supposed truth of the antecedent in ($A_{11(in)}$) *connects* us *indirectly* with a real world, for the fact that the antecedent is true *simpliciter* implies that time has already passed over t_5, but this is still short of speaking the *tensed* language *before* we interpret **e** as, say, *raining in Graz*, and t_5 as, say, 29 March 2000, and even if we interpreted **e** and t_5 in such a way, we could not know what is the date in the given world, for it can be *any* after 29 March 2000. It is interesting, however, that God *could* say exactly *what* is the date in the given world, if he knew which atomic sentences are true *simpliciter* or false *simpliciter*, and which are possibly true *and* possibly false.[24] This is how God could also find a *real difference* between *two* real worlds – if there were two (something we did not presuppose but also did not preclude!) – which with passage of time show no difference with respect to anything that goes on in them *except* that whatever happens in one of them happens only later in the other.[25]

A reference to *tensed* languages (ordinary or formal) may help us here to be more precise about the *meaning* of tensed truths whose existence is said to be implied by the indeterministic axiom (that is formulated tenselessly). Ordinary language contains tensed verbs, while a formal tensed language contains tense operators. There are various ways in which truths expressed in a tensed way are expressed (or tried to be expressed) tenselessly. For instance, one may use the so-called *tenseless present* and *dates* instead of *tensed verbs*. However, the question of the existence of *tensed truths* can be raised *independently* of a given *language* in which they are expressed. A fact (or truth) can be said to be *essentially tensed* (independently of how it is expressed) *if* the sentence through which it is expressed doesn't have the same truth value on all time intervals (i.e., dates). Now, given the validity of the indeterministic axiom, any atomic sentence of the form $\mathbf{AE}(T_n)$ (where **E** stands for **e**, **e**′, **e**″, …, and T for a time constant, i.e., date) expresses such a truth, because it holds, for any such sentence, that on some intervals (dates) it is *neither* true *simpliciter nor* false *simpliciter*, while on some other intervals (dates) it *is* either true *simpliciter* or false *simpliciter*. Similarly, given the validity of the indeterministic axiom, any sentence of the form $\Diamond\mathbf{AE}(T_n)$ also expresses an *essentially tensed* fact, for it holds, for any such sentence, that it is necessarily true on some intervals and only possibly true on some other intervals, i.e., it *can* happen that it is true on some intervals and *false* on some others.

Now, *detensers* may protest by insisting that sentences of any of the two forms, $\mathbf{AE}(T_n)$ and $\Diamond\mathbf{AE}(T_n)$, are *incomplete*, so that they only *seem* to be *essentially tensed* because of their *incompleteness*. Doesn't my *own* formalism and its interpretation allows the *completion* of such sentences by prefixing them with a time constant $[T_m]$, so that the allegedly *tensed* facts become *tenseless* once again? One may reply that such a completion just *shows* that what follows after the prefix is a *tensed* fact, for the truth of the prefixed sentence *depends*, *inter alia*, on the prefix itself, so that *the truth value changes from time to time*. But there is also a major point. It consists in the fact that, given the validity of the indeterministic axiom, the completion that would make $\mathbf{AE}(T_n)$ true *simpliciter*, and either $\Diamond\mathbf{AE}(T_n)$ or $\Diamond\neg\mathbf{AE}(T_n)$ false, doesn't *always* render a sentence that is itself either true *simpliciter* or false *simpliciter*, but *only* on intervals that are *later* than T_n (and, *under certain conditions*, on intervals that *overlap* or are *included in* T_n). *A fortiori*, a completion that would make *all atomic sentences* either true *simpliciter* or false *simpliciter* would be possible only on an interval later than all the intervals that are the elements of our basic set – and so definitely *not from within* any real world segment. So, after all, the existence of *essentially tensed* truths depends on *essential incompletability of tensed truths up to the tenseless ones*.

Let us clarify the last point in detail. There is no reason not to allow the iteration of *temporal prefices* For instance, given that $t_5 \prec t_7$ and that $t_7 \prec t_9$, $[t_9][t_5](\Diamond\mathbf{Ae}(t_7) \wedge \Diamond\neg\mathbf{Ae}(t_7))$ is *true*, as well as $[t_5](\Diamond\mathbf{Ae}(t_7) \wedge \Diamond\neg\mathbf{Ae}(t_7))$, where $[t_9](\Diamond\mathbf{Ae}(t_7) \wedge \Diamond\neg\mathbf{Ae}(t_7))$ is *false*. Namely, the first formula is to be taken to say that it *holds* on t_9 that it held on t_5 that it is possible that **e** occurs on t_7 as well as that **e** doesn't occur on t_7, which is *true* just because $[t_5](\Diamond\mathbf{Ae}(t_7) \wedge \Diamond\neg\mathbf{Ae}(t_7))$ is *true*, whereas $[t_9](\Diamond\mathbf{Ae}(t_7) \wedge \Diamond\neg\mathbf{Ae}(t_7))$ is *false*, because, on t_9, it is *no more possible* that **e** occurs on t_7, if it didn't, or that **e** doesn't occur on t_7, if it did. The problem arises, however, when we turn to the question of the completion of $\Diamond\mathbf{Ae}(t_7)$ and $\Diamond\neg\mathbf{Ae}(t_7)$ *themselves* (on which the completion of $\Diamond\mathbf{Ae}(t_7) \wedge \Diamond\neg\mathbf{Ae}(t_7)$ clearly depends). Namely, there is no problem about their truth values on times *earlier* than t_7, since, given the validity of the indeterministic axiom, the sentences like $[t_5]\Diamond\mathbf{Ae}(t_7)$ and $[t_5]\Diamond\neg\mathbf{Ae}(t_7)$ are *always* true *simpliciter*, and, *a fortiori*, they are *necessarily* true. However, the sentences like $[t_9]\Diamond\mathbf{Ae}(t_7)$ and $[t_9]\Diamond\neg\mathbf{Ae}(t_7)$ are only *contingently* true (and *never* both), depending on what happens on t_7. Now, since, according to the axiom (A_6) (which says that $(t_m)(\exists t_n)t_m \prec t_n$), there is *no* time that is the *latest* time, there must always be sentences that are *incomplete*, in the sense that their *time dependent truth values* can (and in view of *one* of any two concrete sentences of the form $\Diamond\mathbf{AE}(T_n)$ and $\Diamond\neg\mathbf{AE}(T_n)$ *must*) change, according to something that is *still to happen*. This is the point at which the detensers' attempt to complete all tensed facts up to the tenseless ones fails definitely.

It is also important to notice that, though under the validity of the indeterministic axiom there are no atomic sentences that are *necessary a priori*, there are still atomic sentences that are true in *just one possible world* because they are sentences *necessary per accidens*. This *medieval* concept is, however, just something that can be used for the clarification, and *not* something *presupposed*. It is *not* a *presupposition* but a *consequence* of our *standard* assumptions that there is such a thing as a *necessity per accidens*. Namely, if a sentence is true *simpliciter* (i.e., in a real world segment), its negation is no longer possible, and that's why the sentence *is necessary*. But the sentence was not true *simpliciter* at *all earlier* times, and that's why it is necessary *only per accidens*. However, an event does *not cease* to be indeterministic by the very fact that it happened, for there was a time at which its "coupling" with an interval on which it occurred was only possible. Detensers view at all events *as if* they had already happened *without* giving good reason for doing so. That is, however, why they cannot differentiate between "*it can be otherwise*" and "*it could have been otherwise*". For the difference between the two becomes possible only under the assumption that the flow of time changes an accessible possible world into a possible but inaccessible world. There is no contradiction in saying that a sentence is possibly true at one time and necessarily false at some later time, given that time really flows, and for an event to be indeterministic it is *sufficient* that its "coupling" with a time interval cannot be said to be necessary on intervals that precede the given interval (and, under some conditions, on intervals overlapping, or being included in, the given interval), *regardless* of whether the event later occurs on the given interval or not.

Though “possibly true on interval t_1” *does not* mean “possibly true forever”, it still does mean *forever* that which it means, i.e., “possibly true on interval t_1”.

The lesson learnt through the comparison of the deterministic and indeterministic worlds can be expanded to cases where the real world is presupposed to be neither completely deterministic nor completely indeterministic. It is clear that in such a case neither of the two axioms (($A_{11(d)}$) and ($A_{11(in)}$)) holds, but we can equally well use the *conditions* cited in them for distinguishing deterministic and indeterministic events. Deterministic events are those for which the *consequent* of ($A_{11(d)}$) is true, whereas indeterministic events are those for which the *consequent* of ($A_{11(in)}$) is true. But it is very important to notice that such a “mixture” of deterministic and indeterministic events would also require the *flow of time*. The *tenseless* theory of time requires *complete* determinism.

Finally, I want to stress once again that the above analysis is not *circular*, for it has not *presupposed* either the *tensed* theory or the *tenseless* theory of time. It only *turned out* in the end that determinism, in view of the “core” meaning of the term “determinism”, together with a set of standard and minimal presuppositions concerning the rest of the apparatus of the analysis, is not just *compatible* with the *tenseless* theory, but that this theory *implies* (complete) determinism. Hence, we obtained the truth of the tensed theory of time only as a *consequence* of the desire to leave room for indeterministic events to occur in the real world (leaving it to other disciplines to decide whether they really do).

APPENDIX

Definitions of $\wr$, $\cap$ *and* $\subset$ *via* $=$ *and* $\prec$

For the sake of generality, the abutment, overlapping and inclusion relations, denoted by $\wr$, $\cap$ *and* $\subset$ respectively, should be introduced as primitives, but in view of the *linear* system to be sketched below, they can be defined *via* = and $\prec$ as follows:

$$t_m \wr t_n \Leftrightarrow_{\text{def.}} t_m \prec t_n \wedge \neg(\exists t_l)(t_m \prec t_l \wedge t_l \prec t_n),$$

i.e., t_n abuts t_m if and only if t_m precedes t_n and there is no interval between t_m and t_n;

$$t_m \cap t_n \Leftrightarrow_{\text{def.}} (\exists t_l)(\exists t_k)(t_l \prec t_n \wedge \neg t_l \prec t_m \wedge t_m \prec t_k \wedge \neg t_n \prec t_k),$$

i.e., t_m and t_n overlap (on the right side of t_m and on the left side of t_n) if and only if there is an interval preceding t_n but not t_m, as well as an interval following t_m but not t_n;

$$t_m \subset t_n \Leftrightarrow_{\text{def.}} \neg t_m = t_n \wedge (t_l)(t_l \cap t_m \Rightarrow t_l \cap t_n),$$

i.e., t_m is (properly) included in t_n if and only if t_m and t_n differ and no subinterval of t_m lies outside t_n (because there is no third interval that overlaps with t_m without overlapping with t_n) (where $k = 1, 2, \ldots$; $l = 1, 2, \ldots$; $m = 1, 2, \ldots$; $n = 1, 2, \ldots$).

The above three definitions will not all be plausible in every system implicitly defining a time topology. So, for instance, in a branching-time system it will not be generally true that t_m and t_n overlap if there is an interval preceding t_m but not t_n as well as another interval following t_n but not t_m, for t_m and t_n can lie on different branches.

Comments and explanations of the meaning of the axioms of the interval-based system implicitly defining standard time topology

(A_1) $(t_n)\neg(t_n \prec t_n)$

The meaning of and necessity for the introduction of this axiom is evident. Given the difference between = and $\prec$, the fact that each interval is to be identical with itself requires that it cannot precede itself. In view of the above definitions of $\wr$, $\cap$ and $\prec$, also no interval can abut, overlap with or be included in itself.

(A_2) $(t_k)(t_l)(t_m)(t_n)(t_k \prec t_m \wedge t_l \prec t_n \Rightarrow t_k \prec t_n \vee t_l \prec t_m)$

This axiom implicitly defines linearity. There are just three possible outcomes of the cross-comparison of the members of any two pairs of intervals in view of the $\prec$ relation: either the first member of the first pair precedes the second member of the second pair whereas the first member of the second pair does not precede the second member of the first pair, or the first member of the second pair precedes the second member of the first pair whereas the first member of the first pair does not precede the second member of the second pair, or both the first member of the first pair precedes the second member of the second pair and the first member of the second pair precedes the second member of the first pair.

(A_3) $(t_m)(t_n)(t_m \prec t_n \Rightarrow t_m \wr t_n \vee (\exists t_l)(t_m \wr t_l \wedge t_l \wr t_n))$

This abutment axiom precludes the possibility of "gaps" between different non-overlapping intervals, for some two different non-overlapping intervals are either already in the abutment relation or there is a third interval which "connects" them in such a way that this third interval abuts one of the two whereas another one abuts it.

(A_4) $(t_k)(t_l)(t_m)(t_n)(t_k \wr t_m \wedge t_k \wr t_n \wedge t_l \wr t_m \Rightarrow t_l \wr t_n)$

This abutment axiom claims the identity of any abutment, for it says that if two intervals both abut some third interval, it is impossible that there is an interval such that only one of the two abuts it.

(A_5) $(t_k)(t_l)(t_m)(t_n)(t_k \wr t_l \wedge t_l \wr t_n \wedge t_k \wr t_m \wedge t_m \wr t_n \Rightarrow t_l = t_m)$

While (A_3) claims, *inter alia*, that there is an interval connecting two different non-abutting and non-overlapping intervals, (A_5) claims that there is exactly one such interval.

(A_6) $(t_m)(\exists t_n)\ t_m \prec t_n$

(A_7) $(t_m)(\exists t_n)\ t_n \prec t_m$

The reason for introducing the last two axioms is obvious: Standard time topology is such that there is neither beginning nor ending times.

(A_8) $(a_m)(\exists a_n)\ a_n \subset a_m$

This is the density axiom adjusted for our interval-based system: For any time interval (however small), there is an interval that is (properly) included in it.

The above axioms define implicitly *linearity*, *infinity* and *density* of time. However, though "gaps" between intervals are precluded, and although historically continuity meant just the abutment holding between entities of the same dimension, axioms (A_3)–(A_5) together with axiom (A_8) are insufficient to guarantee that the set of all intervals makes up a continuum in the Cantorian sense. For a set to make up a *continuum* in the Cantorian sense, the set should be *both* perfect and coherent (*zusammenhängend*),[26] meaning not only that there are no "gaps" one would detect by "running" over the elements of the basic set itself, but also that one would inevitably fail to "interpolate" a "new" element "between" the elements of the basic set. For instance, the set of all rational numbers does not make up a continuum, the reason being that though any member of the set is an accumulation point of an infinite number of elements of the set, there are accumulations of infinitely many rational numbers which do not have elements of the set as their accumulation points (which is the fact that makes it possible to "interpolate" new elements, such as $\sqrt{2}$ and π). Similarly, if we take all intervals between different rational numbers to be elements of the basic set of a corresponding relational structure, they would not build up a continuum, because there are intervals that do not√ belong to the basic set, such as the interval stretching between $\sqrt{2}$ and π, which can be "interpolated" without disturbing linearity, infinity and density of the original structure.

The quantificational logic used so far, while strong enough for the formulation of the infinity and density axioms, is too weak for the formulation of the continuity axiom. The reason is simply that infinity, contrary to axioms (A_6)–(A_8), where it was only implied, must now be explicitly mentioned at our starting point. The weakest language in which this can be done is $L\omega_1\omega$ that allows for building formulae with an infinite number of conjuncts but where instead of using an infinite number of quantifiers we should introduce an infinite sequence in the metalanguage. For the sake of clarity and simplicity, we shall also introduce four new variable letters: *u*, *v*, *w*, *x*, and define $\succ$ as the inverse of $\prec$. Now, the following (A_9') and (A_9''), taken together, make the basic set coherent:[27]

(A_9') For any infinite sequence of intervals $t_1 t_2 \ldots t_i \ldots$ ordered by $\prec$, it holds that

$$(\exists u)(\wedge_{i<\omega}\, t_i \prec u) \Rightarrow (\exists v)\ (\wedge_{i<\omega}\, t_i \prec v \wedge (w)(\wedge_{i<\omega}\, t_i \prec w \Rightarrow \neg(\exists x)(x \prec w \wedge \neg x \prec v))$$

i.e., if an infinite sequence of intervals ordered by $\prec$ has an upper bound at all, there is an interval amongst the intervals from the basic set that is its lowest upper bound.

(A_9'') For any infinite sequence of intervals $t_1 t_2 \ldots t_i \ldots$ ordered by $\succ$, it holds that

$$(\exists u)(\wedge_{i<\omega}\, t_i \succ u) \Rightarrow (\exists v)\ (\wedge_{i<\omega}\, t_i \succ v \wedge (w)(\wedge_{i<\omega}\, t_i \succ w \Rightarrow \neg(\exists x)(x \succ w \wedge \neg x \succ v))$$

i.e. if an infinite sequence of intervals ordered by $\succ$ has lower bound at all, there is an interval amongst the intervals from the basic set that is its supreme lower bound. Notice that, if an upper (a lower) bound exists, the lowest upper (supreme lower) bound is represented by the equivalence class of an uncountable number of intervals having the same "beginning" ("end").

ACKNOWLEDGEMENTS

I am thankful to all the participants in the discussions at my talks in Pittsburgh (on 12th February, 1999; Center for the Philosophy of Science), in Graz (on 29th March, 2000; Vereinigung für wissenschaftliche Grundlagenforschung, Philosophische Gesellschaft an der Universität Graz und Institut für Philosophie), in Bielefeld (on 12th May, 2000; University of Bielefeld), in Bariloche (on 23rd June, 2000; The Fourth Fellows Conference of The Center for Philosophy of Science in Pittsburgh) and in Karlsruhe (on 12th July, 2000; University of

Karlsruhe), who prompted me to make explicit all the tacit assumptions used in the foregoing argumentation. For the kind invitations to all the talks mentioned I am indebted to James Lennox, Werner Sauer, Martin Carrier and Hans-Peter Schütt. In particular, I have profited very much from discussions that I had with Nuel Belnap, University of Pittsburgh (about the insufficiencies of the tenseless theory of time) and Miodrag Kapetanović, University of Belgrade (about the formulation of the continuity axiom in the language $L\omega_1\omega$). Finally, some important amendments are based on the suggestions of the two anonymous *Erkenntnis* referees.

NOTES

[1] Cf. Smith (1993, I, 2,3).

[2] Cf. Belnap (1992).

[3] In that I shall follow Humblin (1969, 1971), Needham (1981), Burgess (1982) and Bochman (1990).

[4] See Grünbaum (1973, pp. 495ff, pp. 547ff).

[5] *Ibid*., pp. 203ff.

[6] See Arsenijević (1992, p. 181).

[7] Cf. Leibniz (1956, pp. 25–26, 52).

[8] William Newton-Smith traces the Leibnizian requirement back to Aristotle and calls the claim that there are no empty times *Aristotle's Principle*, but he speaks also about the *modal* version of the same principle (see Newton-Smith (1980, 47) according to which possible times are times at which possible events could happen. But to be sure, I do not want to allow for the possibility of a time interval lying between two abutting intervals just because there is a sense in which we might say that an event could happen between two actual events occurring on two abutting intervals. I take, instead, that there must be an interval, empty or non-empty, for it to be possible for an event to occur (or have been occurred) on it.

[9] For the concept of *real possibility*, see Deutsch (1990).

[10] For the concept of a *real world full of different modalities*, see Stalnaker (1976).

[11] David Lewis calls this "necessity in respect of all facts" "fatalistic necessity" (Lewis, 1973, p. 8). But in view of "coupling" events and time intervals, there is no difference between fatalism and determinism, given that we restrict ourselves to the "core" meaning of the terms, which is the only meaning that interests us here. In some other contexts, a more specific meaning of the terms may play the central role. For instance, a compatibilist (someone who holds that our concept of freedom is not incompatible with determinism) may require a specific kind of determination for the truth of his claim, and preclude the compatibility of freedom with pure fatalism.

[12] Let us suppose that, on interval t_m, where $t_m \prec t_n$, there are two accessible possible worlds in which $\mathbf{Ae}(t_n)$ is true, one in which, in addition, $\mathbf{Ae}'(t_n)$ is true, the other in which, in addition, $\neg\mathbf{Ae}'(t_n)$ is true. But, on t_m, either $\Box\mathbf{Ae}'(t_n)$ is true and $\Box\neg\mathbf{Ae}'(t_n)$ false, or $\Box\neg\mathbf{Ae}'(t_n)$ is true and $\Box\mathbf{Ae}'(t_n)$ false. So, either the world in which $\neg\mathbf{Ae}'(t_n)$ is false is *inaccessible*, or it is the world in which $\mathbf{Ae}'(t_n)$ is false. Consequently, there cannot be two accessible possible worlds in which, in addition to $\mathbf{Ae}(t_n)$, $\mathbf{Ae}'(t_n)$ and $\neg\mathbf{Ae}'(t_n)$ are true, respectively. For the similar reason, there cannot be some other two accessible possible worlds in which $\mathbf{Ae}(t_n)$ is true together with some other atomic sentence and its negation, respectively, and, in general, there cannot be two accessible possible worlds at all in which $\mathbf{Ae}(t_n)$ is true, but at most one, the one which is a real world segment.

[13] See Lewis (1986).

[14] Cf. Lukasievicz (1920).

[15] In that respect, our general approach was Hilbertian, as can be seen from the analogy with a well-known case. The "core" meaning of the concept of *straight line* is based on the concepts of *point* and *distance* because straight line connects in the shortest way any two points lying on it. However, there are straight lines and

straight lines – Euclidian, Riemannian, and Lobaczewskian – which differ only as a result of being *implicitly defined* by different sets of axioms. Analogously, the "core" meaning of the diamond- and box-operators is based on the concept of *accessibility*, but its meaning may vary from case to case depending on *which* possible worlds, and *for what* reason, are considered to be accessible, and this can be *implicitly defined* by this or that set of axioms.

[16] Cf. Mellor (1998, 3.2).

[17] Cf. Smart (1980).

[18] Cf. Beer (1994, pp. 91–93).

[19] Cf. Paul (1997, pp. 62ff).

[20] Cf. Smith (1993, II. 5.1).

[21] On one hand, if a real world branches, two contradictory sentences can both be true *after* the branching point (each in one of the two real worlds), so that both sentences are possibly true before the branching point. On the other hand, two real worlds can come across, producing an event that should be considered indeterministic *from within* either of the two. Notice that our account does not preclude any of these possibilities but deliberately does not make any use of them. As for the *fission*, we *did* require that before it happens the two resulting worlds must have the *same modal status* if the event occurring after the fission is to be indeterministic, but the assumption that both worlds are *real* is unnecessarily strong. The much weaker assumption, that both worlds are only accessible possible worlds, fulfilled the requirement as well. However, nothing essential changes if *both* worlds resulting form a fission happen to be real. As for the *fusion*, our account leaves it open whether the event resulting from a fusion should be considered deterministic or not. It is deterministic if the fusion itself was necessary, and indeterministic if the fusion was not necessary. *Miracles* themselves might be said to happen necessarily as well as accidentally. If we deny that this way of speaking about miracles make sense (i.e., reject making sense of *inter-worlds* modalities), we should then accept that the event resulting from a fusion is indeterministic.

[22] Thus Mellor can get that time does not flow but still has direction (see Mellor (1998, 10.2 and 11)).

[23] In Le Poidevin (1991) it is explicitly admitted that "the future cannot be *ontologically indeterminate*", but only "*epistemologically indeterminate*" (p. 130). By contrast, see Rescher (1968). Though we did not speak about the *future* explicitly, any time after some interval t_n can be said to be *the future time relative to* t_n, and that time *is*, according to the indeterministic axiom, *ontologically* and *not only* epistemologically *indeterminate* on t_n, since *no* possible world lying in *the future time relative to* t_n has an *ontologically privileged* status on t_n.

[24] But though in determining dates in a real world God is assumed to be outside it, he is not assumed to be outside time, as in Oaklander's example, where he is just looking at all facts in the world but does not take into account in-the-world-inherent modalities (cf. Oaklander (1994, p. 326).

[25] Cf. Mellor (1998, pp. 19ff.), where, due to the absence of in-the-world-inherent modalities, the same example is used to show that there is no real difference between the two worlds.

[26] See Cantor (1962, p. 194).

[27] This represents a new way of introducing *coherence*, this time in the weakest possible way, within the language $L\omega_1\omega$.

REFERENCES

Arsenijević, M.: 1992, 'Eine aristotelsche Logik der Intervalle, die Cantorsche Logik der´ Punkte und die physikalischen und kinematischen Prädikate: I–II', *Philosophia naturalis* **29**, 1992/2.

Beer, M.: 1994, 'Temporal Indexicals and the Passage of Time', in L. N. Oaklander and Q. Smith (eds), *The New Theory of Time*, Yale University Press.

Belnap, N.: 1992, 'Branching Space-Time', *Synthese* **92**.

Bochman, A.: 1990, 'Concerted Instant-Interval Semantics: I–II', *Notre Dame Journal of Formal Logic* **31**.

Burggess, J. P.: 1982,'Axioms for Tense Logic. II: Time Periods', *Notre Dame Journal of Formal Logic* **23**.

Cantor, G.: 1962, *Gesammelte Abhandlungen*, Hildesheim.

Deutsch, H.: 1990, 'Real Possibility', *Nous* **24**.

Grünbaum, A.: 1973, *Philosophical Problems of Space and Time*, Reidel.

Hamblin, C. L.: 1969, 'Starting and Stopping', *The Monist* **24**.

Hamblin, C. L.: 1971, 'Instants and Intervals', *Studium generale* **24**.

Le Poidevein, R.: 1991, *Chance, Cause and Contradiction: A Defence of the Tenseless Theory of Time*, St. Martin Press, NY.

Leibniz, G. W.: 1956, *Leibniz-Clarke Correspondence* (trans. and ed. H. G. Alexander), Manchester University Press.

Lewis, D. K.: 1973, *Counterfactuals*, Harvard University Press.

Lewis, D. K.: 1986, *On the Plurality of Worlds*, Basil Blackwell.

Lukasiewicz, J.: 1920, *O determinizmie*, *Z zogadnien logiki i filozofii*, Warszawa.

Mellor, D. H.: 1998, *Real Time II*, Routledge.

Needham, P. (1981), 'Temporal Intervals and Temporal Order', *Logique et Analyse* **24**.

Newton-Smith, W. H.: 1980, *The Structure of Time*, Routledge and Kegan Paul. Oaklander, L. N.: 1994, 'Thank Goodness It's Over', in L. N. Oaklander and Q. Smith (eds), *The New Theory of Time*, Yale University Press.

Paul, L.: 1997, 'Truth Conditions of Tensed Sentence Types', *Synthese* **111**. Rescher, N.: 1968, 'Truth and Necessity in Temporal Perspective', in R. Gale (ed.), *The Philosophy of Time*, Humanities Press, New Jersey.

Smart, J. J. C.: 1980, 'Time and Becoming', in Peter van Inwagen (ed.), *Time and Cause*, Reidel.

Smith, Q.: 1993, *Language and Time*, Oxford University Press. Stalnaker, R.: 1976, 'Possible Worlds', *Nous* **10**.