Miloš Arsenijević

The Future Sea Battle and Performing an Infinite Task: Two Remarkable Cases Concerning the Logicist Thesis

Abstract: In his 2018 article which concerns normativity of natural language reasoning, Smokrović deals with the alleged incongruence between descriptive nature of formal logic and normativity of reasoning in everyday argumentation, juridical debates or scientific, philosophical and even mathematical dialogues. Contrary to the radical stance, according to which there is no possible normative use in the reasoning which is sensitive to propositional content that should be interpreted in connection to the real world, Smokrović supports what he calls logicist thesis, according to which in any case in which there is a discrepancy between standard logic and some argumentation that we consider correct, we ought to explain why it is so and find an appropriate form, i.e. a logic more or less close or remote to standard logic, through which the correctness of the given argumentation could be vindicated. In this article I analyze, following the requirements of the logicist thesis, two remarkable and intriguing philosophical debates, which concerns 1) the future contingences and the problem of logical determinism, and 2) the impossibility of performing infinite tasks.

Key words: logicist thesis, Aristotle, future contingencies, logical determinism, the principle of bivalence, temporal-modal logic, infinite tasks, Grünbaum, anti-infinitism, the incompossibility of remote possible worlds.

In his 2018 article "Informal Reasoning and Formal Logic: Normativity of Natural Language Reasoning" Nenad Smokrović deals with the recently raised question concerning the *relation* and *alleged incongruence* between *descriptive nature* of formal logic, which supposedly concerns purely syntactic relations between propositions in view of the preservation of truth within classical propositional and predicate calculi, and *normativity of reasoning* in everyday argumentations as well as in juridical debates or scientific, philosophical and even mathematical dialogues (p. 457).

The debate started with Harman's (1986) radical view, according to which there is no possible normative use of formal logic in everyday reasoning, which is sensitive to *propositional content* that should be interpreted in connection to the *real world*. Even in the most simple and obvious cases, such as the following one cited by Smoković, the necessity of truth preservation is allegedly not secured, in spite of the fact that the argument is put in one of the classically valid forms.

(*p*) (According to the time-table) *The 8 a.m. bus from Rijeka to Zagreb starts either from platform 1 or from platform 2;*
(*q*) (Actually) *The bus does not start from platform 2;*
(*C*) (Therefore) *The bus starts from platform 1.*

What if, for any reason whatsoever, platform 1 is not available at 8 a.m.? Then, the bus will not start form platform 1 either. So, in such a case, though both premises are true, and though we would hardly say that the reasoning was incorrect, the conclusion is false. The point is that even if in many (or even all) previous cases the conclusion was true, it was not *necessarily* true, while according to formal logic it *should be* so. This should be enough for claiming that there is a kind of discrepancy between validity of formal logic and similar forms of reasoning in everyday argumentation.

Reacting to Harman's radical stance, MacFarlane and Hartry Field (cf. Smoković, ibid., pp. 460-463) suggested different variants of the so-called *bridge principle* between formal logic and everyday reasoning that should give normative force to classically valid logical forms, thus enabling their application in everyday reasoning. Without going into formal details and differences between the ways in which the bridge principles are formulated, let us explain informally how, according to Varga, Stenning and Martigton (2015), appropriate deontic reading of the logical scheme applied in the above example enables us to say that the reasoning in passing from the first two premises, *p* and *q*, to the conclusion *C*, was correct even if *C* turns out false.

Let us suppose that, by looking at the time-table, we have seen by our own eyes that the 8 a.m. bus to Zagreb starts from platform 1 or platform 2. Let us suppose, in addition, that, by passing by platform 2, we have found that, today, the bus on platform 2 is not the bus going to Zagreb but to Pula, and that the bus driver himself has confirmed that, in order to catch the bus to Zagreb, we ought to go to platform 1. There has also been no information that anything has been changed in relation to what stands in the time-table. And then, only much later, we learn that, today, there is no bus to Zagreb. What has happened? Something *abnormal*! So, on one hand, our reasoning can be said to be *correct* just because *C* has turned out false only due to something *abnormal*, while, on the other hand, what has happened *has happened regardless* of whether it is normal or abnormal. The fact that something abnormal has happened does not make our reasoning incorrect in spite of the fact that *C* is false.

In other cases, the explanation of the seeming discrepancy between the validity of a given logical form used in the argumentation and the possible incorrectness of the argumentation may happen to be explainable in some other ways, but anyway, it should be explainable in *some* way. On the basis of this, Smoković argues in favour of what he calls *logicist thesis*, according to which in *any* case there should be *some* logic, more or less close or remote to standard logic, through which the correctness of argumentation could and should be vindicated.

Varga, Stenning and Martington have argued that in the given example the logic that can resolve the problem is the *default logic*: $p \wedge q \rightarrow C$ can be used for saying that we correctly pass from the justified belief that p and q are true to the belief that C is true, if *C is* true *by default*, i.e., false *only if* something *abnomal* has happened.

One might be tempted to think that the *default logic* could be incorporated in standard logic if the form of the above reasoning, as Varga, Steanning and Martigton suggest, were schematized in the following way (see Smoković 2018, p. 467):

$$p \wedge q \wedge \neg \mathbf{ab} \rightarrow C$$

where $\neg$**ab** means that *nothing abnormal is the case*. But it is not so. In the first place, **ab** does not stand for any standard proposition as p and q do, but is rather a schematic formulation standing for an indefinite number of propositions. There is no way to extensionalize $\neg$**ab**. Even if we could add an infinite disjunction to $p \wedge q$ instead of $\neg$**ab**—which is not possible to do in standard first order propositional calculus—we would not get that the addition of anyone of the disjuncts alone can make $p \wedge q \vdash C$ wrong, since, for any A, *if* $p \wedge q \vdash C$ is valid, so is *also* $p \wedge q, A \vdash C$. In order to get $p \wedge q \vdash C$ *valid* and $p \wedge q, A \vdash C$ *non-valid*, we need some of Anderson and Belnap's *systems of the logic of relevance*, where $A, B \vdash C$ may be non-valid in spite of the fact that A $\vdash C$ supposedly is (Anderson and Belnap 1975).

After all, I accept Smoković's *logicist thesis*, according to which in any case in which there is a discrepancy between standard logic and everyday argumentation which we supposedly consider correct, we ought to explain why it is so and find an appropriate *logical form* in accordance with which the given argumentation will be correct. Now, as it is said in the introduction, Smoković believes that the *logicist thesis* should hold not only in everyday and juridical debates, but also in scientific, philosophical and even mathematical dialogues. In view of this, in what follows, I will analyze, in brief, some intriguing philosophical debates I have been dealing with for a rather long period of time, in order to find out whether, in order to settle the dispute, we need to depart from standard logic (or mathematics), and if it turns out that we have to, what are the ways in which we could or should do it.

Example 1: Aristotle's Future Sea Battle

Curiously enough, the first philosopher to be confronted with the problem concerning the *logicist thesis* was the father of traditional logic, Aristotle himself (*De interpretatione* 19 a 23).

If an utterance of the sentence "There will be a sea battle tomorrow" is true, then it seems that it is *determined* that there will be a sea battle tomorrow. For otherwise, how could the utterance be true? If, however, an utterance of the sentence "There will be a sea battle tomorrow" is false, then it seems that it is *determined* that there will be no sea battle tomorrow. For otherwise, how could the utterance be false? Thus, it seems that it is *determined* whether there will be a sea battle tomorrow or not – and so for any other future event whatsoever. This, however, is in conflict with the plausible assumption that at least for some possible future events, it is not predetermined whether they will take place or not.

The kind of determinism implied by the fact that whether something will happen at some time or not is necessarily predetermined by the truth of the corresponding propositions—that it will happen and that it will not happen respectively—is called by Schlick (1931, p. 202) *logical determinism*. Logical determinism is stronger than any other sort of determinism, and as such threatens any interpretation of quantum mechanics that implies indeterminism. But independently of this, it is incompatible with everyday reasoning, according to which what will happen supposedly depends on our deliberations and decisions connected with various coincidences in the real world.

The purpose of this example is not to raise the debate about determinism and indeterminism. The point is only to show that there seems to be an obvious discrepancy between everyday reasoning about future contingencies and obeying some of the basic logical principles. So, according to the *logicist thesis*, it seems that, if we want to preserve the validity of everyday reasoning, we have to abandon, change or restrict some of the logical principles. Which one?

It is clear that for Aristotle it cannot be the *principle of contradiction*, for any contradiction (ἀντίφασις) whose form is $p \wedge \neg p$ is always necessarily *false*, whatever proposition p stands for. Whatever some interpreters may think, for Aristotle it is also not the *principle of excluded middle*, for he put it explicitly that $p \vee \neg p$ is always *true*, because the very form in which the affirmation and the opposite negation stand here secures the truth, and so also even if neither of the two (οὐ…τόδε ἢ τόδε) is already (ἤδη) either true or false. But then, given that it is necessary that $p \vee \neg p$ will be true tomorrow, it could and should be said to be true today as well. However, neither p nor $\neg p$ themselves may happen to be true or false at some earlier time, independently of the fact that $p \vee \neg p$ is true. So, what has been much later called by Łukasiewicz the *principle of bivalence* (Łukasiewicz 1922, p. 126) is that whose universal validity was challenged and after that restricted by Aristotle with the use of the *Sea Battle* example.

Aristotle's reaction is deeply revisionist in view of the logic that he has established. Namely, it prevents the very formalization of propositional calculus in which, by using the standard definitions of $\wedge$, $\vee$ and $\neg$, the *principle of bivalence* is derivable from the *principle of contradiction* and the *principle of excluded middle*. Namely, if it holds, for *any* proposition, that the conjunction of it and its negation is always false, while the disjunction of it and its negation is always true, then it follows that *every* proposition must have one and only one of the two truth values—*truth* or *falsity*—which is exactly what the *principle of bivalence* claims. So, under Aristotle's revision of his own logic, being either true or false ceases to be a necessary condition for being a proposition and the revised logic contains *truth-value gaps*.

It was only Łukasiewicz who recognized clearly that the *Sea Battle* was directed against one of the *basic* and mutually *independent* logical principles, which he called, in his famous *Rector's Speech* 1922, the *principle of bivalence*. But he reacted in a different way to the problem of the *logicist choice* between alternatives that the *Sea Battle* had imposed. Instead of restricting the *principle of bivalence*, he constructed the *three-valued logic system* (Łukasiewicz 1918, 1920), which, instead of truth-value gaps, contains, in addition to *truth* and *falsity*, the third value: *indeterminacy*. While *true* propositions are about something that *is* and *false* propositions about something that *is not*, the propositions with the third truth value are about something that does not have a real correlate but which is yet *possible*.

Łukasiewicz's three-valued and many-valued logical systems represent a nice piece of formal and philosophical logic, and one may be tempted to think that in order to make the everyday reasoning concerning future contingences correct, there is hardly a better way to formalize it. But it should be noticed that in Łukasiewicz's formalization *time* as such does not play any role, while in the *sea battle* challenge we are dealing with not just a *possible* sea battle, but with a *future* sea battle. So, there are those, to whom I myself belong, who think that we can still save the *principle of bivalence* when trying to make room for everyday reasoning about future contingences *if* we formulate a logical system in which both *time* and *modalities* are simultaneously taken in account. This is done in the system of *temporal-modal* logic of events TM, whose semantics is given in my 2016 article, whereas the complete axiomatization of it is formulated in a not yet published manuscript, presented at my visiting talk at the University of Siegen on 19th Dec. 2019 (Arsenijević and Jandrić, forthcoming, 2023). I will summarize what the solution to the *sea battle* problem looks like according to TM.

TM is formalized within the interval-based system of the time continuum, where t_1, t_2, …, t_n, … are constants standing for particular time intervals and t_1, t_2, …, t_n, … variables ranging over the set of all time intervals. Now, the elementary well-formed-formulae of TM is any $\mathrm{E}(\mathrm{t}_n)$, where t_n is replaceable by any time-constant or time-variable and E stands for any elementary event (those which happen uninterruptedly) e_1, e_2 ,…, e_n, …, as well as any formulae preceded by a quantifier ∀ or a quantifier ∃, or by temporal operators $\{\mathrm{t}_1\}$, $\{\mathrm{t}_2\}$, …, $\{\mathrm{t}_i\}$ … or $\{t_1\}$, $\{t_2\}$, …, $\{t_i\}$, …, or modal operators ◊ or □. Temporal and modal operators can be iterated and combined.

Now, the endless interval-based time continuum in any TM model contains two abutting parts, one real, meaning that on any of its intervals something has happened, and the other, imaginary, on whose intervals nothing has yet happened. The intervals of the first one are called full or actualized, the intervals of the second one empty or non-actualized. For temporal operator $\{\mathrm{t}_i\}$ and any formula A, $\{\mathrm{t}_i\}$A is true if and only if t_i is actualized and A true at it, which is the case if A is a logical truth or any of the factual truths about what happened or failed to happen on t_i, as well as any of all the truths about what happened on intervals that precede it (which are, hence, actualized themselves). But it is important to notice that it can be not only any of the truths about what happened on any of the actual intervals that happened earlier but also about any of the actual intervals that ended later, which seems to lead us back to logical determinism. Yet, I will shortly explain why it is not the case.

In the general semantics of modal logic, formula □A is said to be true if and only if A is true in all accessible possible worlds, and ◊A is taken to be true if and only if there is an accessible possible world in which A is true. Now, in the standard possible world semantics, the truth of a formula prefixed by a modal operator is assessed from a single world, and, therefore, it is not necessary to point to the world from which the accessible worlds are accessible. However, in the system TM, the real world always contains *an infinite number of actual worlds* (because the real world consists of an infinite number of actualized worlds), so that some possible worlds are accessible from some actual worlds but not from others. If, for instance, an event e happened on an interval t_n, then on an earlier interval t_m it was possible for e not to occur on t_n, while on t_n itself this possibility is precluded. Thus, there is a merely possible world, in which e does not happen on t_n, which is accessible from the world actualized on t_m but not from the world actualized on t_n.

So, the formulae with a modal operator outside the scope of a temporal one lack a determinate truth value, as in such cases it is not specified which actual world's set of accessible possible worlds is to be taken into account. In TM we can meaningfully talk about possibilities only by bearing in mind what has, up to a certain time, already been actualized. So, the status of the formulae such as, for example, $\Box e(t_n)$ and $\Diamond e(t_n)$, should be understood by the analogy to the well-formed but open formulae in predicate logic, which become definitely true or false only after some further qualification. Formulae with iterated modalities can, accordingly, be true or false only if the sequence of modal operators is, as a whole, subjected to a temporal operator.

Hence, we can also speak of merely possible worlds being accessible from other merely possible worlds but only provided that the first merely possible world in the chain is accessible from some actual world. In other words, the talk of possible possibilities, possible necessities, etc., has to be *anchored* in the real world.

Let us now, in view of what has been previously said, turn to the *sea battle* problem. Let us take that t_m precedes t_n, but so that t_m is actual and t_n non-actual. This is the case when t_m refers to *today* and t_n to *tomorrow*. Let $e(t_n)$ denote *the sea battle that happens tomorrow*. Then, according to the above definition of the truth of $\{t_i\}A$ for any A, $\{t_m\}e(t_n)$ is *false* and $\{t_m\}\neg e(t_n)$ *true*, since on t_m there is no actualized world in which $e(t_n)$ is true. But now, the point is that although $\{t_m\}\neg e(t_n)$ is true, $\{t_m\}\{t_n\}\neg e(t_n)$ is *false*, since for $\{t_n\}\neg e(t_n)$ to be true it is necessary that t_n is actual and $\neg e(t_n)$ true on it, which, not being the case, renders $\{t_m\}\{t_n\}\neg e(t_n)$ *false*. In TM, the prefixing of a temporal operator is generally not a trivial matter, since it may affect the truth value of the ensuing complex formula. This is exactly the fact through which *logical determinism* is avoided, for the fact that *there will be the sea battle tomorrow* means that it is true today that it will be true tomorrow that the sea battle happens that day, which is expressed by $\{t_m\}\{t_n\}e(t_n)$, which is *false*.

Let us now remember the seemingly threatening case in which $\{t_m\}e(t_n)$ may be true even if t_n is later than t_m. This can be so *only if* some world is already actualized on t_n. For only then, either $\{t_n\}e(t_n)$ or $\{t_n\}\neg e(t_n)$ is true. So, what matters here is, in the first place, whether t_n is actual or not, and then, if it is, whether $e(t_n)$ is true on it or not.

Completely in accordance with ordinary language, it is not only false today that the sea battle will happen tomorrow—$\{t_m\}\{t_n\}e(t_n)$—but also that it will not happen—$\{t_m\}\{t_n\}\neg e(t_n)$. Both being false, $\{t_m\}\{t_n\}e(t_n)$ and $\{t_m\}\{t_n\}\neg e(t_n)$ are not contradictory but only contrary, and that is why both $\{t_m\}\Diamond e(t_n)$ and $\{t_m\}\Diamond\neg e(t_n)$ may be true. Today, it is both possible that the sea battle happens tomorrow as well as that it does not happen. But this does not mean that at t_m it is true that on t_n it will be both possible that the sea battle happens and that it does not happen, since $\{t_m\}\{t_n\}(\Diamond e(t_n) \wedge \Diamond\neg e(t_n))$ is *false*. However, though today it is only possible that the sea battle will happen tomorrow (as well as that it will not happen), if the sea battle really happens tomorrow, it will be true tomorrow that it was true the day before that it would be true the day after that the sea battle had happened that day. Similarly, if the sea battle does not happen tomorrow, it will be true tomorrow that it was true the day before that it would be true the day after that the sea battle had not happened that day.

Given that TM models are supposedly distributed along one and the same time continuum, the model in which *today* refers to t_n contains, as a part, the model in which it refers to t_m but is not a mere extension of it in view of just the factual truths, for there is an infinite number of ways in which it was possible that the real world history could have developed from the state in which *today* referred to t_m and the state in which it refers to t_n. All the possibilities are preserved as *modal truths* about what *could have been the case*. So, history is much richer than the set of factual truths. The system TM enables us not only to speak about what *is* and what *can be* the case but also about what *could have been* the case. All this also explains why in ordinary language there is an asymmetry between prediction and retrodiction.

The reason why we cannot know the truth about the future sea battle is not a matter of *epistemological* fact. Namely, in the given case, the impossibility of knowledge is completely based on the *logico-ontological* fact that *there is nothing to be known*, since there are different possible ways that lead to this or that outcome. We may *guess* that it will be so-and-so, but to guess is not to know. Everything depends on deliberations, decisions and coincidences of events that are not yet actual. However, once the sea battle really happens or really fails to happen, there is just *one single path* the history has paved to this, and we can explain (in principle at least) *how* it has come into being by taking into account *actual* deliberations, decisions, coincidences, etc.

If there is just one real world history in view of any given apex as the boundary between the real and the imaginary part of the time continuum, there must be *one privileged model* from the equivalence class of isomorphic models that represents the real world in view of a given apex. Though it could have been otherwise, what is, necessarily is. The distribution of elementary events over the *real* part of the time continuum is *necessarily such as it is*. So, for any given instant, there must be a *unique* real world history that ends at it. Then, though any instant is represented through an equivalence class of models regardless of the history that has paved the way to it, there is always a privileged world line that represents *the* history of *the* real world up to the given instant.

The *factual* truths about events in a model which represents only a part of the history are *preserved* in the privileged model that describes the whole real world history up to the given instant. Then, the *development* of the real world history can be viewed as a continuous transition from one privileged model to others such that each of them represents the real world history up to a certain instant *as if* it ended at that instant. *The so-called flow of time* is nothing else but such a continuous transition from one privileged model to others as a consequence of the development of the real world history, where each of previous models represents an earlier development of the real world history up to a certain instant.

The model in which t_n is actual contains not only factual truths of previous models but also truths about all the possibilities in previous models. If it was possible yesterday that it would rain today, $\{t_m\}\Diamond e(t_n)$, it remains true today that it was possible yesterday that it would rain today even if $\{t_n\}\neg e(t_n)$ has become true.

All these facts are in accordance with intuition and ordinary language. In view of the logicist thesis that we are investigating, we can conclude that the crucial questions concerning the *sea battle* can be dealt with within standard classical predicate logic after the introduction of temporal and modal operators and the suitable choice of the system of axioms. The fact mentioned above, that in TM formulae with iterated modalities can have a determinate truth value only if the sequence of modal operators is, as a whole, subjected to a temporal operator is in accordance with everyday language, where we speak of modalities from within the real or imaginary segments of one and the same time continuum. As Nuel Belnap puts it, "If a certain possibility is real, […], it must be part and parcel of *Our World*" (Belnap 2007, p. 87, n. 2), so that "the brilliantly conceived doctrine of Lewis 1986 (and elsewhere) ought to be rejected".

Example 2: Impossibility of performing infinite tasks

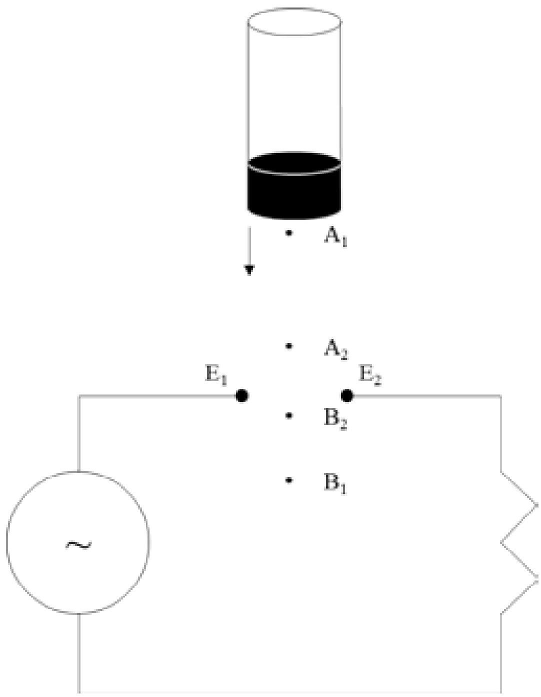
In *Physics* 233 a 22 and 263 a 8, Aristotle mentions a variant of Zeno's second kinematic paradox, constructed by an unknown author, where the runner, in order to reach the goal, has to count distances that become smaller and smaller according to the geometric progression ½, ¼, … As he takes it for granted that it is not possible to finish counting to infinity, Aristotle takes the given example as an argument that a continuum, be it spatial or temporal, does *not consists of actual* parts that could be counted. But what if, independently of the question concerning the structure of the continuum, the spatial and/or temporal parts individuated by a geometric progression are actually physically distinguished, so that, in order to reach the goal, the runner de facto has to perform the task consisting of an infinite number of steps?

In my *Analysis* paper (Arsenijević 1989) I dealt with the question of whether a limited space can contain an infinite number of physically distinguished parts. Here, we shall deal with the temporal variant of the problem, where the steps of an infinite task are sufficiently well distinguished by the very acts of their performance. At the middle of the last century, this question was discussed by a considerable number of well-known philosophers and mathematicians, who can be divided, independently of additional differences, into two groups: infinitists (Taylor 1951, Watling 1952, Maxwell and Feigl 1961, Grünbaum 1968, 1969, Salmon 1975) and anti-infinitists (Hilbert 1926, Weyl 1949, Black 1951, Wisdom 1952, Schwayder 1955, TeHennepe 1963, Chihara 1965, Hilbert and Bernays 1968). In the context of this paper that concerns Smokrović's *logicist thesis*, we are interested in the relation between the *arguments* used in the debate and *logical forms* in which they are to be put, in order to see whether logic as such can be of use in trying to settle the dispute between the two parties. This will also concern debates in the philosophy of mathematics, which, as mentioned in the introduction, may be involved in the question about *descriptive nature* of formal logic and *normativity of reasoning*.

As the case study, we shall combine two famous examples: *The Thomson Lamp* (Thomson 1968) and *The Staccato Run* (Grünbaum 1968, Arsenijević 1988). The Thomson lamp is just an ordinary lamp, except that it may be on for 1 sec, off for ½ sec, on for ¼ sec, and so on ad infinitum. If it changes its states in such a way, is it on or off after 2 sec elapse? The *staccato run* of a runner is the motion in which he stops at the half-way point after ½ sec, rests there for ½ sec, moves further on for ¼ sec with the same speed, when he stops and rests for ¼ sec, and so on ad infinitum. Where will he be after 2 sec?

Benacerraf remarked in his illuminating 1962 paper that the given description of the way in which the Thomson lamp functions concerns only its being on and off *within* the *open* interval of 2 sec. Nothing is said about its state *after* this time. Both being on as well as being off after 2 sec are *compatible* with what was happening within the open interval of 2 sec. After all, the lamp can be destroyed just as 2 sec elapse, so that it makes no sense to say that it is either on or off.

Taken as such, Benacerraf's remark is correct. It *suggests* that, from the mathematical point of view, the performance of the infinite task is possible. However, in order to *prove* that it is physically possible, Grünbaum (1968, p. 97) imagined an electric device in which the state of the lamp *after* 2 sec should be *predictably on as a consequence* of an infinite number of jabbing motions of the button of a lamp. If it is so, then the given infinite task would be proved to be completable, since it would have a direct physical consequence.



Let the button of a lamp be equipped with an electrically conducting base which can close the circuit by fitting into the space between the exposed circuit elements E_1 and E_2 (see the diagram). Let an infinite process begin so that the button, whose base is at A_1, 1/2 cm above E_1E_2, is pressed down as to reach in (1/2 + 1/4) sec the point B_1, 1/4 cm bellow E_1E_2. After being at rest for 1/4 sec, the button is raised in (1/4 + 1/8) sec up to the point A_2, 1/8 cm above E_1E_2, being at rest there for 1/8 sec. Then, the button is pressed down in (1/8 + 1/16) sec as to reach the point B_2, 1/16 cm bellow E_1E_2, being at rest there for 1/16 sec. And so on, and so forth, let the process of downward and upward motions of the button be continued endlessly, by being successively in positions A1, B1, A2, B2, A3, B3, and so on. Now, it can be argued that, if electric device remains intact after 2 sec, the base of the button of the lamp can be neither above nor bellow E_1E_2, so that it must be *at* E_1E_2, which means that the lamp will be *predictably on*. This proves, according to Grünbaum, that, under the given description, it is not only possible that within the open interval of 2 sec the infinite task consisting of downward and upward jabbing motions is *performable* but also that it is *completable*, since it has a definite physical consequence. In the given case, the performance of an infinite task leads to the performance of a *super-task*.

So, at least in the given case, the infinitists seem to be right. However, Allen Janis, Grünbaum's colleague at the University of Pittsburg (see Grünbaum 1968, p. 101, n. 64), concocted the situation ingeniously by suggesting an alternative switching arrangement, where the circuit is *closed* if the button base is at E_1E_2 after having come there from above but *open* if it is at E_1E_2 after having come there from bellow (due to some isolator which covers the button base automatically after it passes through E_1E_2 from above and being removed automatically after it passes through E_1E_2 from bellow). Under this switching arrangement, the lamp should be both on and off, or neither on and off, after 2 sec have elapsed, since the button base should be at E_1E_2 by coming there both from above and from bellow. So, the super-task should be feasible under Grünbaum's original arrangement but not feasible under Janis's alternative arrangement, though the two infinite processes are kinematically identical.

At the conference in Bielefeld 1994, in Grünbaum's presence, I drew attention to the problem that in Janis's arrangement there is a contradiction as the consequence of the fact that there is no last jabbing motion, either from above or from below, which is also the case in the original arrangement, where the infinite task is allegedly feasible. Surprisingly, Grünbaum reacted sympathetically and said that the addition of new conditions can make an otherwise feasible process unfeasible. He added that even his original arrangement may seem suspect due to specifically dynamical difficulties in effecting the infinitude of accelerations, because in ever shorter time intervals of upward and downward motions the acceleration should increase (and decrease) boundlessly. In relation to this question, he pointed to his 1969 paper, where it is shown how the last difficulty can be obviated through Richard Friedberg's re-arrangement, in which intermittent motions can proceed at suitable decreasing average velocities such that both successive peak velocities and accelerations during the decreasing subintervals converge to zero as we approach the terminal instant. It seems, however, that Grünbaum missed the point of Janis's re-arrangement.

Janis's re-arrangement is not intended to show how *some* change in the original description can make the originally feasible process unfeasible, but rather to suggest that, given that the introduction of the isolator is something external which does not influence the kinematics of the process so that the continuation of it is feasible up to any point within the open interval of 2 sec, the paradoxical outcome after 2 sec elapse casts doubt to the feasibility of the *kinematically identical* infinite process in the original arrangement. Namely, if one of the two kinematically identical processes is feasible, the other one should also be feasible, and if one of the two is unfeasible, the other one should be unfeasible as well So, the fact that in the re-arrangement the button base cannot admittedly be at E_1E_2 after 2 sec elapse, it cannot be there in the original arrangement either.

It is important to notice that the outcome in the second case is not analogous to the case in which the apparatus would be destroyed just as 2 sec elapse, for in such a case it would make no sense to ask whether the lamp is on or off. In Janis's re-arrangement the button base *should be* at E_1E_2 but the lamp cannot be *neither* on *nor* off. It is not so because of the presence of the isolator at E_1E_2 after 2 sec have elapsed, for this would make the circuit open. It is so because the button base didn't reach E_1E_2 from bellow. Similarly, the circuit is not closed not because of the absence of the isolator at E_1E_2, but because the button base didn't reach E_1E_2 from above. The conclusion is that after 2 sec the paradoxical outcome is the consequence of the fact that the button base should have reached E_1E_2 *neither from above nor from bellow*. But then, after 2 sec the button base could not be at E_1E_2 in the original arrangement either.

The dialectic of the above debate makes the problem extremely tricky. On one hand, it is hard to see what else but the impossibility to perform an infinite task can prevent the button base, in any of the two cases, to be at E_1E_2 after 2 sec elapse. It is highly implausible to assume that the difference between the two arrangements is based on the fact that the apparatus "knows" in advance if the outcome would be paradoxical or not, so that it performs the infinite task in the original arrangement only, given that the task is performable in both cases up to any of A_n and B_n points. On the other hand, given that the process in both cases is performable up to any of A_n and B_n points, the question is where the button base can be after 2 sec elapse but at E_1E_2.

So, after all, if we reasonably accept that the impossibility to perform the infinite task in Janis's re-arrangement means that it is impossible to perform it in the original arrangement as well, the fact that the infinite process is performable up to any of A_n and B_n points should not imply that it can be finished by reaching E_1E_2 when 2 sec elapse. If the process continues ad infinitum, 2 sec *will not* elapse, and if 2 sec elapse, the process *had to stop* developing further on after some point. If $f(A_n)$ and $f(B_n)$ means that the process is performed up to A_n and B_n respectively, then $\forall_n \Diamond f(A_n)$ and $\forall_n \Diamond f(B_n)$ are true, but $\forall_n f(A_n)$ and $\forall_n f(B_n)$ false.

In many modal logic systems $\forall_n \Diamond f(A_n) \rightarrow \Diamond \forall_n f(A_n)$ is not a theorem, and there is a lot of examples in finite models that may illustrate this. For instance, if there are too many pieces of good food on the table, each of them can be eaten during the party, but not all. In our case the truth of $\forall_n \Diamond f(A_n)$ lies in the dynamic character of the process which can develop without end. But it can be developing without end only because it is not unconditionally true that 2 sec will elapse. By dealing with Grünbaum's thought experiment, we smoothly mixed the extremely remote possible world in which the number of downward and upward jabbing motions increases boundlessly within the open interval of 2 sec with the everyday situation in which it seems obvious that the time interval of 2 sec must elapse unconditionally. We overlooked the possibility that the two possible worlds, one in which the process will develop endlessly and the other one in which 2 sec will elapse may be incomposable, and therefore incompossible.

However, the fact that the two worlds are incomposable does not mean that we must choose from the very beginning *which* is the world we are speaking about. We may allow both that the process *may* develop boundlessly *and* that 2 sec *may* elapse, and leave it open whether the process will continue endlessly *or* 2 sec will elapse. What we mustn't suppose simultaneously is that the process *will* develop endlessly and that 2 sec *will* elapse.

Conclusion

The analyzed examples show how we ought to proceed, following the requirement of Smokrović's logicist thesis, in order to obtain logical forms applicable, respectively, to our ordinary way of speaking about future contingencies and to the argumentation concerning the problem of the possibility to perform an infinite task. Fortunately, it has turned out that in both cases we do not have to depart from standard predicate logic but only to extend it appropriately.

In the first of the two cases, as expected, we have to introduce temporal and modal operators, since we want to speak about possible future events. After the suitable selection of temporal and temporal-modal axioms, we get a system in which neither the *principle of bivalence* nor *the principle of excluded middle* is restricted, but in which, in perfect congruence with everyday reasoning, it is nether true that it will be true tomorrow that the sea battle will happen nor that it is true that it will be true tomorrow that the sea battle will not happen. But, if the sea battle happens tomorrow, this will mean that it is not possible any longer that it did not happen, for "what is, necessarily is, when it is, and what is not, necessarily is not, when it is not", as Aristotle put it. This is why in this case it will become true tomorrow that it was true the day before that it would be true the day after that the sea battle had happened that day. At the same time, and again in accordance with everyday reasoning, though it ceased to be possible that the sea battle has not happened, it remains true that it could have been otherwise. And finally, and again in accordance with everyday reasoning, in which the prediction-retrodiction asymmetry plays an important role, the unpredictability of future contingencies is not a matter of epistemological but of logico-ontological fact that there are many and in principle innumerable ways in which the world history can develop, while there is just one, privileged way in which it has de facto developed.

In the second example, the comparative analysis of Grünbaum's and Janis's arrangements of the electric device—in which the button base of Thomson's lamp is allegedly at E_1E_2, after an infinite number of downward and upward jabbing motions—shows that, if an infinite process is feasible in one of the two arrangements, it must be so in the other one too. But this leads to a contradiction, since the outcome is contradictory in Janis's arrangement. So given that the presence of the isolator is something external that makes no difference from a kinematical point of view, we need nothing but standard logic to conclude the unfeasibility of an infinite task

So far, so good! But this is not the end of the story, since the argument against the infinitism does not give the answer to the question concerning the position of the button base after 2 sec *elapse* and the whole device remains *intact*. It seems that, according to standard mathematical analysis, the button base can be just nowhere but at E_1E_2. However, it is standard mathematical analysis itself which, with the help of modal logic, gives a solution. Namely, within the open interval of 2 sec the infinite process is *performable* endlessly, but this does not mean that it is *completable* within the closed interval of 2 sec. For the 2 sec interval to be closed, there must be the last jabbing motion, which would be possible only if the process ceases to develop endlessly at some point within the open interval of 2 sec. So, there are two possible worlds, one in which the process develops endlessly, and the other one in which it ceases doing that. Since the two worlds are *incompossible*, it is *not unconditionally* true that 2 sec will elapse, which seems odd only because the first one is *extremely remote* from the second one, which is the world in which we live. It is very interesting that David Hilbert, one of the greatest mathematicians of twentieth century, who proclaimed emphatically that nobody will push us out from Cantor's paradise, claimed that infinite tasks are physically unfeasible (Hilbert 1926 and Hilbert and Bernays 1968, p. 16). What he missed to say is that this is not *accidentally* so but a *consequence* of the *incompossibility* of the two worlds, one in which the interval of 2 sec is *open* and the other one in which it is *closed*. The same holds for the so-called remarkable curves, which though mathematically definable are not drawable, not because they are two-dimensional, but because they are non-differentiable at one point at least, which the curve cannot approach from any direction whatsoever (see Arsenijević 1994).

So, after all, we have found *logical forms* which are in *congruence* both with *everyday reasoning* and *mathematical analysis*.

References

Anderson, A. R. and Belnap, N. 1975. *Entailment—The Logic of Relevance and Necessity*. Princeton University Press.

Aristotle 1831. *De interpretatione*. In: *Opera*, ed. I. Bekkeri. Walter de Gruyter.

Aristotle 1831. *Physica*. In: *Opera*, ed. I. Bekkeri. Walter de Gruyter.

Arsenijević. M. 1988. "Solution of the Staccato Version of the Achilles Paradox". In: A. Pavković (ed.). *Contemporary Yugoslav Philosophy: The Analytic Approach*. Kluwer (pp. 27- 55).

Arsenijević. M. 1989. "How many physically distinguished parts can a limited body contain?". *Analysis* 49 (1) (pp. 36-42).

Arsenijević. M. 1994. "Mathematics, Infinity and the Physical World". *Dialektik* 3 (pp. 89-107).

Arsenijević. M. 2016. "Avoiding Logical Determinism and Retaining the Principle of Bivalence within Temporal Modal Logic: Time as a Line-in-Drawing". In: S. Gerogiorgakis (ed.). *Time and Tense*. Munich: Philosophia Verlag (pp. 7-37).

Arsenijević. M. and Jandrić, A. 2023. (forthcoming) "The Arrow and the Flow of Time in a Non-Deterministic World According to the Temporal-Modal System TM". *Logic and Logical Philosophy.*

Belnap, N. 2007. "An Indeterministic View of the Parameters of Truth". In: T. Müller (ed.) *Philosophie der Zeit*. V. Klostermann (pp. 87-113).

Benacerraf, P. 1962. "Tasks, Super-Tasks and Modern Eleatics". *Journal of Philosophy* 59 (24) (pp. 765-784).

Black, M. 1951. "Achilles and the Tortoise". *Analysis* 11 (5) (pp. 91-101).

Chihara C. S. 1965. "On the Possibility of Completing an Infinite Process". *Philosophical Review* 74 (pp. 74-87).

Grünbaum A. 1968. *Modern Science and Zeno's Paradoxes*. London: George Allen and Unwin.

Grünbaum A. 1969. "Can an Infinitude of Operations be Performed in a Finite Time?" *British Journal for the Philosophy of Science* 20 (pp. 203-218).

Harman, G. 1986. *Change in View: Principles of Reasoning.* MIT Press.
Hilbert, D. 1926. "Über das Unendliche". *Mathematische Ananalen* 95 (pp. 161-190).
Hilbert, D. and Bernays, P. 1968. *Grundlagen der Mathematik*. Springer Verlag. Berlin (1st ed. 1934).
Łukasiewicz, J. 1918. Farewell Lecture by Professor Jan Łukasiewicz, delivered in the Warsaw University Lecture Hall on March 7, 1918. In: *Jan Łukasiewicz – Selected Works*. Amsterdam: North-Holland, 1970 (pp. 84-86).
Łukasiewicz, J. 1920. On Three -Valued-Logic. In: idem, *Selected Works*. Amsterdam: North- Holland, 1970 (pp. 87-88).
Łukasiewicz, J. 1922. On Determinism. In idem, *Selected Works*. Amsterdam: North-Holland 1970, (pp. 110-128).
Maxwell, G. and Feigl, H. 1961. "Why Ordinary Language Needs Reforming". *Journal of Philosophy* 58 (18) (pp. 488-498).
Salmon, W. C. 1975 *Space, Time, and Motion*. Encino CA: Dickenson.
Schlick, M. 1931. "Die Kausalität in der gegenwärtigen Physik". *Naturwissenschaften* 19 (pp. 145-162).
Smoković, N. 2018. "Informal Reasoning and Formal Logic: Normativity of Natural Language Reasoning". *Croatian Journal of Philosophy*. Vol. XVIII, No. 54 (pp. 455-469).
Schwayder, D. S. "Achilles Unbound" *Journal of Philosophy* 52 (pp. 449-459).
Taylor, R. 1951. "Mr. Black on Temporal Paradoxes". *Analysis* 12 (2) (pp. 38-44).
TeHennepe, E. 1963. "Language Reform and Philosophical Imperialism: Another Round with Zeno". *Analysis* (Suppl.) 23 (pp. 43-49).
Thomson, J. F. 1968. "Tasks and Super-Tasks". *Analysis* 15 (1) (pp. 1-13).
Varga, A., Stenning, K., and Martignon, L. 2015. "There Is No One Logic to Model Human Reasoning: the Case from Interpretation". In: U. Farbach and C. Shon (eds.). *Proceedings of the first workshop on bridging the gap between human and automated reasoning*. Berlin (pp.32-46).
Watling, J. 1952. "The Sum of an Infinite Series". *Analysis* 13 (2) (pp. 39-46).
Wisdom, J. O. 1952. "Achilles on Physical Racecourse". *Analysis* 12 (3) (pp. 67-72).
Weyl, H. 1949. *Philosophy of Mathematics and Natural Sciences*. Princeton University Press.