Time, Tense, and Reference

edited by Aleksandar Jokić and Quentin Smith

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10

Real Tenses

Miloš Arsenijević

10.1 Introduction

Viewed historically, the so-called real world, understood as that which could be and should be thought of as it is *in itself*, independently of how it *appears* to us, passed through and is still subjected to numerous "ontologico-philosophical purgatories," which have, in one way or another, eliminated various entities and properties that are normally (i.e., pre-philosophically) believed to be its components. Some of the "purifications" seem unquestionable, some are less convincing, some are problematic or at least disputable, and some seem too radical to be acceptable.

Let us recall a few well-known examples. At present, it is hardly questionable, even among nonphilosophers, that *being beautiful* is neither a property of the physical world nor of its parts as such, since, contrary to, say, being of such and such a size or such and such a shape, something or somebody is beautiful only if and when it, he, or she appears to us as such. It is less certain whether the physical world is really colorless, as it should be after having passed through Democritus' "atomist purgatory." According to the "new materialist purgatory," the world is an even more deserted place: the real world as a whole is void of the mental qua mental after all mental processes have become identified with brain processes. On the other hand, Berkeley's "idealist purgatory" produced a world that is completely nonmaterial. Parmenides' "ontological purgatory" has probably been the most comprehensive of all (*pace* Gorgias' ontological nihilism!): it has left the real world without any heterogeneity, plurality, and change.

Time is one of the entities that has been "burnt" more than once, but still managed to "survive" in one gestalt or other. Contrary to Newton's substantivalist view of time, Leibniz's relationalist conception of time has made time dependent on changes and reducible to certain kinds of relations between events (see Newton 1953, p. 17, and 1972, p. 46; Leibniz 1956, pp. 25–26, 52). But having ceased to be an entity existing per se, Leibniz's time has emerged as a property of the changing world. "Leibniz's world" is not timeless: time is real just to the extent to which a certain order between events—the temporal order—is real. Kant's "transcendental purgatory" has been more substantial: Kant has rejected both Newton's view that time is an entity in its own right (which could exist even if nothing else did) and Leibniz's view of time as just the temporal order between events, so that time has become only a *mode* of our knowledge of objects and ceased to be anything ascribable to things in themselves. (see Kant 1911, I.2, secs. 4–7). McTaggart's "neo-Hegelean purgatory" has been even more merciless: understood in whatever manner, time has turned out to be something *contradictory in itself* (see McTaggart 1908, pp. 457 ff.).

Here, I will reexamine a contemporary, very popular, and somewhat merciful purgatory to which time is subjected, which I will call "*Mellor's purgatory*" in honor of one of its most prominent initiators, Hugh Mellor.[1] "Mellor's world" is *not timeless*, but it is void of some properties that are pre-philosophically believed to be primary temporal properties: its *pastness*, *presentness*, and *futurity*. In brief, "Mellor's world" is *tenseless*. Of course, tenses are claimed to be unreal not as features of verbs and sentences, but as features of what sentences (and thoughts expressed by them) are about.

In what follows, I shall take time in "Mellor's world" to be Newtonian or Leibnizian at will, since I do not think that the difference between the two could affect the main argument. But I'll cite in an endnote the variants of deterministic and indeterministic axioms that the Leibnizian conception would require. As for the difference between Newtonian and Einstein-Minkowskian time (see Minkowski 1923), it will not influence the arguments, because we can choose to deal with contested properties in relation to one particular place in the world.

10.2 McTaggart's Proof

As previously mentioned, McTaggart believed that reality must be timeless, because the concept of time, after his analysis, turned out to be contradictory in itself. McTaggart's analysis tacitly assumes that reality should be considered timeless if it is tenseless, and that, at the same time, the world history can be represented by simply placing particular events onto the one-dimensional time axis. The explicit assumption is that *being past*, *being present*, and *being future* are mutually incompatible properties.

Let us consider a segment of what McTaggart calls the B-series, where four events, e_1, e_2, e_3, and e_4, occur successively at some given place and are brought into one-to-one correspondence with four abutting time intervals on the one-dimensional time axis, t_1, t_2, t_3, and t_4, respectively. Then, if the four events are future events that should once become present and then past—and that is what situates them in what McTaggart calls the A-series—each of them must in turn possess mutually incompatible properties of *being future*, *being present*, and *being past* with no change in their fixed position on the one-dimensional time axis, which is absurd.

As it stands, McTaggart's proof is a valid argument, so that, in order to avoid the implication that reality is timeless, one should question at least one of the premises.

Mellor, and all those who are popularly called 'detensers', rejects the assumption that if reality is tenseless, it is *eo ipso* timeless. They rather state the objectivity of the B-series (or various B-series), denying at the same time the objectivity of any A-series.

Some detensers, though not all of them, hold that McTaggart's proof is sufficient to vindicate their move toward the tenseless theory of time.[2] I do not share such an opinion. Why shouldn't we have our cake and eat it too by rejecting the second assumption? Why wouldn't the world history between t_1 and t_4 be represented by the two-dimensional matrix in figure 10.1 (see Schlesinger 1994a,b)? The empty, half-empty, and full circles represent future, present, and past events, respectively. The first row depicts the world history at t_1, when e_1 is present; the second row depicts

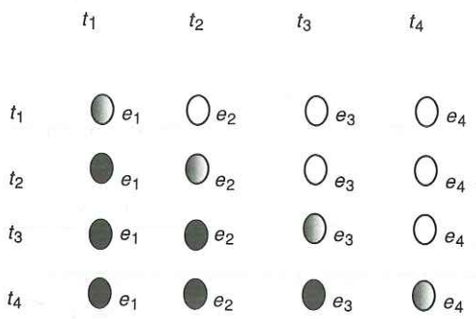


Figure 10.1

the change in the world history resulting from the fact that, at t_2, e_2 is present; the third and fourth rows depict the two consecutive changes in the world history resulting from the facts that, at t_3 and t_4, e_3 and e_4 are present, respectively. The four events $e_1, \ldots, e_4$ are always ordered in the same way, but they are not brought into correspondence with $t_1, \ldots, t_4$ independently of the fact that the world history differs from time to time in view of which event is the present event. Owing to the two-dimensional representation, an event occurring at a particular time *can* be future, present, and past, because it is one thing to say that an event occurs at that time and another thing to say that it is future before that time, present at that time, and past after that time.

10.3 A System of Events Containing Dates and Tenses

Nothing more has been established so far than that the possibility of a two-dimensional representation of the world history prevents detensers' vindication of the alleged necessity to choose between B-series and A-series, that is, between dates and tenses. This doesn't mean that detensers don't have, or couldn't have, other reasons for rejecting the A-series, that is, for rejecting the reality of tenses. The same holds, mutatis mutandis, for those who want to endorse just the A-series and reject the B-series altogether. In this section, I will sketch a *tense logic system* whose interpretation includes both B-series relations and tenses without implying any contradiction or a vicious regress under the two-dimensional representation and the flow-of-time assumption. However, before doing this, I want to be explicit about the main purpose of this chapter as a whole in order to avoid unnecessary commitments concerning the system to be constructed.

Given that McTaggart's proof is not conclusive, it turns out that detensers have two types of opponents, because the anti-detensing program can be shaped as one that is either more or less ambitious. The more ambitious program is that of Prior, developed and comprehensively elaborated from both the semantic and metaphysical point of view in Ludlow 1999. According to Ludlow, the standard B-theory relations 'before' and 'after' can be composed out of more basic A-series relations (p. 126), since every tensed sentence in natural language has either an explicit or an implicit 'when'-clause that serves to do the work of temporal anaphora (pp. 12, 134). So, for instance, by speaking about some past event, stating explicitly the date of its occurrence or not, one doesn't *refer* to a time in the past but simply connects the given event with some other events via a series of 'when'-clauses implicitly given by the context of the utterance. What makes it possible for any such series to be anchored is the fact that indexicals represent an essential part of our tensed language: it is always something *present* that we start or end up with, at least implicitly, when we speak of past and future events. As for the cases in which *reference* to different times seems to be unavoidable, as when we want to say for an event that it happened *more times*, the notion of *times* is simply to be decomposed into *different* sets of 'when'-clauses (p. 128).

Now, the program underlying the tense logic system to be outlined in this section is *less* ambitious, partly because the main purpose of the chapter is to show that the detensing analysis fails at an important (some may claim crucial) point, even if we sympathize with detensers' motives and give credit to their reinterpretation of our tensed language. Namely, though I believe that the tense logic system to be offered is of interest in itself, I will use it, in section 10.4, as a basis for the detensers' reinterpretation of a tensed language, since the system does make use of both dates and tenses. And then, in section 10.5, I will formulate a temporal modal logic of events in a tenseless language in order to show that, in spite of all our sympathies for detensers' motives, the concept of the in-the-world-inherent modalities requires the flow-of-time assumption.

Let us start with an axiomatic temporal system of *intervals* as time's basic stuffs. I find it natural to use such a system, instead of an instant-based one, since the system to be built up on it is a logic of events, and any event lasts for a certain period of time. Individual variables $t_1, t_2, \ldots, t_n, \ldots$ are to be directly interpreted as ranging over the basic set of intervals (as in Hamblin 1971; Needham 1981; Burgess 1982; Bochman 1990), so that intervals are not confined to a metalanguage, as in propositional time logic. However, the time topology will be standard (as I proved in Arsenijević 2003, secs. 2 and 3, one and the same topology can be alternatively defined by an instant-based system and by an interval-based system of axioms). Particular intervals will be denoted by $\mathrm{t}_1, \mathrm{t}_2, \ldots, \mathrm{t}_n, \ldots$, whereas the relation symbols $=, \prec, \{, \cap, \subset$ are to be interpreted as the identity, precedence, abutment, overlapping, and inclusion relations, respectively. The informal reader is asked to understand these relations intuitively, while the formal reader can consult the axioms in appendix A,[3] which define implicitly the relational structure under consideration. The relational structure for which the cited axioms are satisfied is endless, linear, and continuous. The elementary well-formed formulas are $t_1 = t_2$, $t_1 \prec t_2$, $t_1 \{ t_2$, $t_1 \cap t_2$, and $t_1 \subset t_2$, as well as any formulas obtainable by the substitution of t_1 and/or t_2 through some other variable(s) and/or constant(s).

Now, for the sake of argument, we take it that, in our logic of events, $e, e', e'', \ldots$ denote qualitatively and spatially well specified events whose complete individuation is obtained by "pairing" them with particular time intervals that they (completely) occupy. So, for instance, $e(\mathrm{t}_1)$ denotes a qualitatively and spatially well specified event e occurring on interval t_1, while $e(t_1)$ denotes the same qualitatively and spatially well specified event occurring on the interval that t_1 takes as a value by ranging over the set of all intervals.[4]

In order to avoid inessential complications, we shall also take it that events we are dealing with $(e, e', e'', \ldots)$ are qualitatively homogeneous and continuous in themselves, like a position change by the uniform motion. The concept of such events—let us call them *elementary* events—can be easily adjusted to cover various everyday events, such as *uninterrupted raining* or *uninterrupted snowing* (see Arsenijević 2002, sec. 3). As I showed elsewhere, for such elementary events it is reasonable to

stipulate that if e occurs on t_1, it occurs on any subinterval of t_1. In appendix A, this stipulation is expressed through axiom A_{10}.

We have now reached the crucial point. How are tenses to be introduced?

Let A be a sentence-forming operator, so that, for instance, $Ae(\mathrm{t}_1)$ is the sentence claiming that e occurs on t_1. Now, though the given event is supposedly well individuated, not only spatially but also temporally, the information conveyed by $Ae(\mathrm{t}_1)$ is still incomplete, according to the tensers' view presented above, owing to the lack of temporal characterization concerning tenses.

In view of how the system is sketched so far, we cannot use the Priorian strategy (see Ludlow 1999, p. 108) of swapping talk of propositions for talk of events by putting tense operators in front of $Ae(\mathrm{t}_1)$. Namely, if $Ae(\mathrm{t}_1)$ is true, it is true on any time interval, and if it is false, it is false on any time interval, so that an operator in front of $Ae(\mathrm{t}_1)$ would be superfluous. We must treat tenses as *monadic properties of events* and introduce them into our system through *predicate letters* that are to be put *after A*.

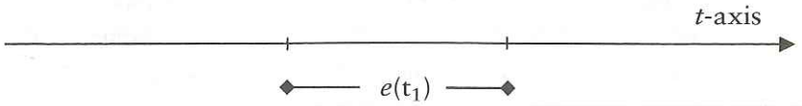
Let N be a predicate letter that is to be read as 'is present'. How is $ANe(\mathrm{t}_1)$ to be understood? First, in our system of intervals there can be no such thing as *absolute presentness*, since the system does not contain either instants or time-minima. Second, the presentness, N, of a qualitatively, spatially, and temporally individuated event, $e(\mathrm{t}_1)$, is *relative* to time intervals on which $ANe(\mathrm{t}_1)$ is supposed to hold.

Before I state the truth conditions for $ANe(\mathrm{t}_1)$, let me introduce the rest of the tense predicate letters. Since any two intervals are in just one of the basic relations (identity, precedence, abutment, overlapping, inclusion), there are more tenses than just the past, present, and future. In particular, there can be eight elementary tenses, which will be denoted by the *tense predicate letters* F, F_N, *F-N*, N, *N-P*, *F-N-P*, N_P, and P, to be read as 'future', 'partly-future-partly-present', 'partly-future-and-present', 'present', 'present-and-partly-past', 'partly-future-present-and-partly-past', 'partly-present-partly-past', and 'past', respectively.

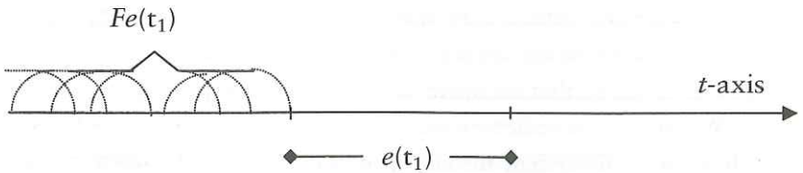
Now, in addition to the elementary well-formed formulas defined above, the atomic sentences are also $Ae(\mathrm{t}_1)$, $AFe(\mathrm{t}_1)$, $AF_Ne(\mathrm{t}_1)$, $AF\text{-}Ne(\mathrm{t}_1)$, $ANe(\mathrm{t}_1)$, $AN\text{-}Pe(\mathrm{t}_1)$, $AF\text{-}N\text{-}Pe(\mathrm{t}_1)$, $AN_Pe(\mathrm{t}_1)$, and $APe(\mathrm{t}_1)$, as

well as all the formulas obtained by substituting for *e* some other letter denoting an event and/or by substituting for t_1 some other constant or a time variable letter. Complex formulas are to be built up by the use of the propositional calculus connectives. Any open sentence, be it atomic or complex, can be closed in the way to be specified below.

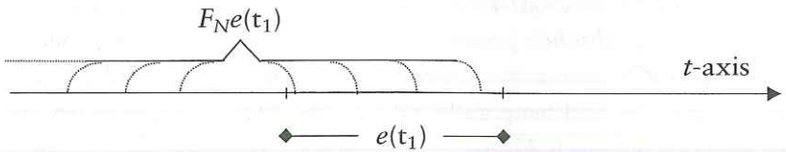
Let us now turn to the truth conditions for atomic sentences.



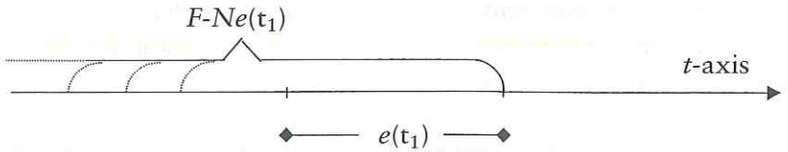
$Ae(t_1)$ is true if and only if the event denoted by *e* occurs on the interval denoted by t_1.



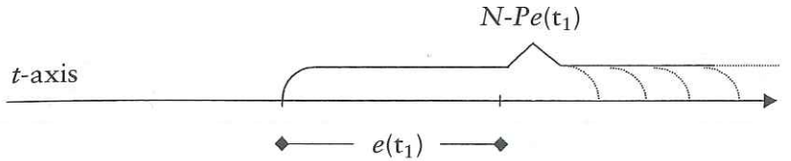
$AFe(t_1)$ is true on, and only on, all the intervals that precede t_1, given that, in addition, $Ae(t_1)$ is true.



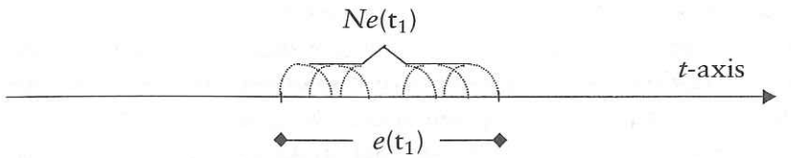
$AF_Ne(t_1)$ is true on, and only on, all the intervals that overlap with t_1, given that, in addition, $Ae(t_1)$ is true (where 'overlap with t_1' means that they overlap with t_1 on its left side).



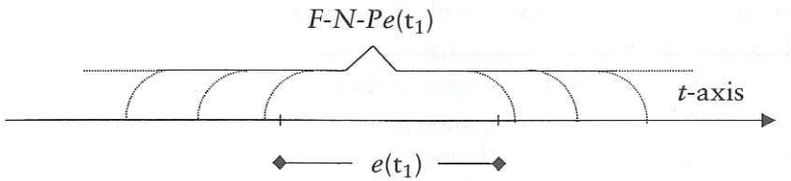
$AF\text{-}Ne(t_1)$ is true on, and only on, all the intervals in which t_1 is included but which do not have any subinterval that is later than t_1, given that, in addition, $Ae(t_1)$ is true.



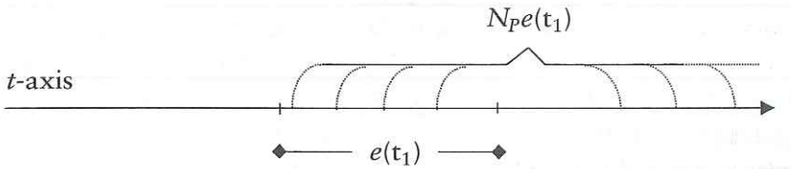
$AN\text{-}Pe(t_1)$ is true on, and only on, all the intervals in which t_1 is included but which do not have any subinterval that is earlier than t_1, given that, in addition, $Ae(t_1)$ is true.



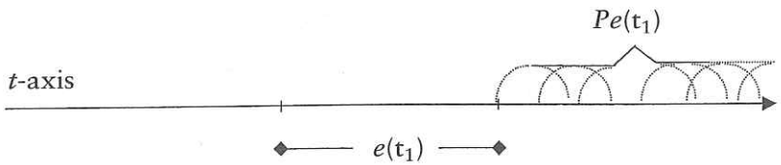
$ANe(t_1)$ is true on, and only on, t_1 as well as all the intervals that are included in t_1, given that, in addition, $Ae(t_1)$ is true.



$AF\text{-}N\text{-}Pe(t_1)$ is true on, and only on, all the intervals in which t_1 is included but which have both subintervals that are earlier than t_1 and subintervals that are later than t_1, given that, in addition, $Ae(t_1)$ is true.



$AN_Pe(t_1)$ is true on, and only on, all the intervals with which t_1 overlaps, given that, in addition, $Ae(t_1)$ is true (where 'overlaps with t_1' means that they overlap with t on its right side).



$APe(t_1)$ is true on, and only on, all the intervals that are later than t_1, given that, in addition, $Ae(t_1)$ is true.

It is easy to see how the above truth conditions are to be used in the case in which a variable letter stands in place of t_1. It should be remembered, however, that none of the atomic sentences (whose truth conditions are given above) is closed, since it is not said, for any of them, on which time intervals it is supposed to hold. So, given that T stands for a tense predicate letter, we stipulate that $[t_2]ATe(t_1)$ says that $ATe(t_1)$ holds on t_2, whereas $(t_n)ATe(t_1)$ and $(\exists t_n)ATe(t_1)$ say that $ATe(t_1)$ holds on all intervals and on some intervals, respectively, where the quantified variable must remain seemingly blind, not occurring explicitly in the rest of the sentence (i.e., in place of t_1).

Let us now consider the closures of $Ae(t_n)$ and $Ae(t_n)$.

$(\exists t_n)Ae(t_n)$ is true if and only if there is a value of t_n for which $Ae(t_n)$ is true. Given the above stipulation—expressed through A_{10}: $(t_n)(Ae(t_n) \Rightarrow (t_m)(t_m \subset t_n \Rightarrow Ae(t_m))$—which restricts all the events we are dealing with to those that we have called elementary, there is never just one interval on which e occurs, if there is any such interval at all. But e can also reoccur after a certain period of time, and there is also no reason for not allowing e to occur throughout time. Thus, $(t_m)Ae(t_m)$ can be contingently true. The crucial question is then, What about the truth of $(\exists t_n)(t_m)ATe(t_m)$ and $(t_n)(t_m)ATe(t_m)$ in such a case?

Given that $(t_m)Ae(t_m)$ is true, $(\exists t_n)(t_m)ATe(t_m)$ is not true, since the order in which the quantifiers are introduced requires that a value for t_n be fixed first, and then it is always possible to find such a value for t_m that the formula under consideration is false. For instance, if t_k is the fixed value for t_n, then $AFe(t_m)$ is false for some t_l (as a value of t_m) such that $\neg t_l \prec t_k$.

Given that $(t_m)Ae(t_m)$ is true, $(t_n)(t_m)ATe(t_m)$ is also not true, since the truth of $(t_n)(t_m)ATe(t_m)$ would imply the truth of $(\exists t_n)(t_m)ATe(t_m)$.

Given that $(t_m)Ae(t_m)$ is true, $(t_m)(t_n)ATe(t_m)$ is not true as well, which is again easy to realize by consulting the truth conditions for any sentence obtainable by substituting a tense predicate for T.

However, given that $(t_m)Ae(t_m)$ is true, $(t_m)(\exists t_n)ATe(t_m)$ is true, since for any t_k as the value of t_m, it is easy to find some t_l as the value of t_n such that $AFe(t_k)$ is true according to the truth conditions for $AFe(t_k)$, and so also, mutatis mutandis, for any other tense predicate standing for T.

Now, since $(\exists t_n)(t_m)ATe(t_m)$, $(t_n)(t_m)ATe(t_m)$, and $(t_m)(t_n)ATe(t_m)$ are all false if $(t_m)Ae(t_m)$ is true, these formulas are also false if $(\exists t_m)Ae(t_m)$ is true, and, a fortiori, if $(\exists t_m)Ae(t_m)$ is false. So, they are never true. At the same time, $(t_m)(\exists t_n)ATe(t_m)$ is true not only if $(t_m)Ae(t_m)$ is true, but if and only if $(t_m)Ae(t_m)$ is true. For if $(\exists t_m)Ae(t_m)$ is true but $(t_m)Ae(t_m)$ false, then the only true closed sentences containing tense predicates are those of the form $(\exists t_m)(\exists t_n)ATe(t_m)$.

However, either in the case in which $(t_m)Ae(t_m)$ is true, or in the case in which only $(\exists t_m)Ae(t_m)$ is true and $(t_m)Ae(t_m)$ false, there is no value of t_n for which any two different formulas obtained from $(t_m)(\exists t_n)ATe(t_m)$ or $(\exists t_m)(\exists t_n)ATe(t_m)$ by substituting different tense predicates for T could be true, as is easy to see by comparing all the truth conditions for tensed sentences. Thus, whether an event is present throughout time or not, it is always (meaning 'on any time interval') *either* future *or* partly-past-partly-present *or* partly-past-and-present *or* present *or* present-and-partly-future *or* partly-past-present-and-partly-future *or* partly-present-partly-future *or* future, where the disjunction is exclusive.

Now, since there are no further relations—in addition to those already envisaged in the above truth conditions for the tensed sentences—that could hold between t_1, occurring in $Ae(t_1)$, and some other time interval, there is no room for the introduction of a new elementary tense: the above exclusive disjunction holding between different tenses is also exhaustive. So, in view of the elementary tenses, there is no way to reveal a contradiction by using a McTaggart-style proof. But what about iterated tenses? Does an iteration of tense predicate letters lead to a vicious regress?

In appendix B, the formal reader can find the recursive definition of the truth conditions for the atomic sentences containing iterated tense

predicates, which simply represents a generalization of the definition of the truth conditions for the sentences containing only one tense predicate. The same appendix contains four lemmas, whose philosophical significance I will now summarize for the informal reader.

Lemma 1 shows that by iterating tenses we never obtain a sentence that is true on all time intervals, which means that no elementary sentence, however many tense predicates it contains, is tenselessly true. That's why our logic system is a *tense* logic system in which we are dealing with *tensed truths*: there is always something else to be said about an event of which it is said that it occurs on an interval, and this additional information can be obtained only through a *tensed* truth.

Lemma 2 says that, given that it is true at all that an event occurs on an interval, any tensed sentence about that event is true on some time interval; that is, no tensed sentence is tenselessly false just because it is tensed. If it is tenselessly false, it is so only because the event said to occur on the given interval does not occur on it.

Lemma 3 implies the exclusiveness of any tense in relation to different tenses of the same complexity: if a tensed sentence about some event's occurrence on an interval is true, no other tensed sentence (about the same event's occurrence) that contains the same number of tense predicates is true at the same time.

Lemma 4 shows that no iteration is trivial—in other words, that any new tense predicate transforms the sentence true on an interval into a sentence false on that interval, except when the new predicate is an *N*-predicate (which is completely in accordance with our intuition that if one says that an event is present (past, future, etc.), one adds nothing nontrivial by saying that its presentness (pastness, futurity, etc.) is present).

Now, all the consequences of the four lemmas are easily and consistently interpretable with the use of the two-dimensional representation and under the flow-of-time assumption. While time has been flowing toward t_1 (or, better, while it has been "producing" intervals that precede t_1), the supposedly true but incomplete information contained in $Ae(t_1)$ is to be completed through $AFe(t_1)$, which is true on all the intervals that precede t_1. Once time has started to "produce" t_1,[5] but still hasn't "produced" it in full, $AFe(t_1)$ ceases to be true, the incomplete information

contained in $Ae(t_1)$ being then truly completable through $AF_Ne(t_1)$ or $ANe(t_1)$, depending on the intervals on which the complete information is supposed to be conveyed, where there is no interval on which $AF_Ne(t_1)$ and $ANe(t_1)$ are both true. (Notice that in the two-dimensional representation, there is no time for which any two of $Fe(t_1)$, $F_Ne(t_1)$, and $Ne(t_1)$ would be members of one and the same row!) The flow of time having "produced" t_1 in full, but nothing more than that, $Ae(t_1)$ is completable through $AF\text{-}Ne(t_1)$ or $ANe(t_1)$, depending again on the intervals on which the complete information is supposed to be conveyed, and where there is again no interval on which $AF\text{-}Ne(t_1)$ and $ANe(t_1)$ are both true. And so on, and so forth. It is easy to see at which point $Ae(t_1)$ becomes completable only through $AF\text{-}N\text{-}Pe(t_1)$, $AN\text{-}Pe(t_1)$, $AN_Pe(t_1)$, or $APe(t_1)$, depending again on the intervals on which the complete information is supposed to be conveyed. It is also easy to see what the explanation would look like if we turned to sentences containing iterated tense predicates.

10.4 *Entia praeter Necessitatem Non Sunt Multiplicanda!*

The above tense logic system together with its interpretation shows that the tensers' view cannot be discredited on the basis of its inconsistency or some other formal insufficiency. But detensers can try to do something much more promising. They can try to reinterpret the meaning of all the tense predicates so that they cease to be interpreted as *monadic properties of events*, being interpreted instead in terms of *basic relations* holding between the times of events' occurrences and the times of the respective sentences' (actual or possible) utterances. If they succeed in doing that, detensers can simply quote Occam's razor and proclaim their view favorable on the basis of this methodologico-ontological principle. For the B-series relations are something already presupposed in the tense logic system outlined above, so that tensed properties of events are entities that, *praeter necessitatem*, ought to be avoided.

There are two points that make such a strategy particularly appealing. First, considering the truth conditions for tensed sentences cited above, we could see that they are tenseless. Second, according to the sketched tense logic system, at least, tensed sentences are something placed in

time; so detensers do not change the starting point by trying to reinterpret tenses in terms of relations between the times of events' occurrences and the respective sentences' (actual or possible) utterances. The only difference should turn out to consist in the fact that *being placed in time* does not mean, according to detensers, to be *tensed in an irreducible manner*.

To regard sentences as entities placed in time is something quite natural. Any sentence (about some event's occurrence) is formulated, uttered, thought of, considered, reconsidered, discussed, taken into account, and so on, at some time, that is, in some time interval. But, of course, a sentence can be utterable (though not actually uttered), entertainable (though not actually considered), formulable (though not actually formulated), and so on. Even then, however, it is utterable (entertainable, formulable, etc.) *on an interval* (or *on any interval*, but, in any case, *in time*). The information content of a sentence may be supposed to depend or not to depend on the time of an (actual or possible) utterance, but the utterance time is, in any case, a *particular time interval*.

The last triviality is of great importance, since the detensers' idea (at least as I am presenting it here) is to take what tensers view as an essential part of the information conveyed and reinterpret it as inessential (and rather global) information—not about any event's property but about the time at which the information about the event's occurrence is conveyed (or would be conveyed if the sentence were uttered).[6] So, for instance, if the information about some event's occurrence is to be conveyed via a future tense sentence, then, according to tensers, an essential part of the information is information about something real, that is, about the futurity of the event (or of the event's occurrence). According to detensers, however, there is no such real thing as futurity, the future tense of the sentence indicating only that the time at which the information is conveyed precedes the time of the event's occurrence.

Similarly, the past tense of a sentence indicates, according to detensers, that the time at which the information is conveyed is later than the time of the respective event's occurrence. The present perfect tense indicates that the time of the respective event's occurrence is included in the time at which the information is conveyed (e.g., the present perfect tense in 'It has been raining today' indicates, in contrast to the simple past tense in 'It was raining yesterday', that the time of the event's occurrence—the "rainy time"—is included in 'today' as time explicitly said to be the time at which the information is conveyed). And so on, and so forth. Remembering all the tense predicates—F, F_N, $F\text{-}N$, N, $N\text{-}P$, $F\text{-}N\text{-}P$, N_P, P—which mirror exhaustively standard relations holding between intervals, we may say that English needs more tenses, but it is often so when we compare different natural languages or natural and artificial languages: one of the languages compared turns out to be either poor or unnecessarily rich (the tense logic system of German, for instance, is even poorer than that of English: it does not differentiate between the simple past tense and the present perfect tense).

Let us see how the detensers' reinterpretation functions when applied directly to the tense logic system outlined in section 10.3. According to the original interpretation, $Ae(t_1)$ conveys *essential* but still *incomplete* information about the otherwise well-specified event e: it says exactly when e occurs, but the lack of a tense predicate leaves one of e's essential properties unspecified. According to the detensers' reinterpretation, $Ae(t_1)$ conveys *essential* and *full* information about e, since the addition of a tense predicate would not give us information about e's property but only (very globally!) about the time at which the sentence is used in a particular case.

It does not follow, of course, that detensers mean that the information conveyed by the use of tense predicates is unimportant. For various reasons, such information can be extremely important. If I do not have a watch and want to cross a bridge announced to be bombed, it is much more important for me to know whether the bridge *was* already bombed or *will be* bombed soon than to know the exact time of bombing. But the *importance of tenses* does not mean simply their *objectivity* or *irreducibility*. Objectively, the bombing is an event taking place either earlier than or later than the time I am asking about it, and it is only my ignorance about the exact time that makes tenses in the given example more important than dates.

Turning to *iterated* tenses, we can generalize the detensers' interpretation in an obvious manner. For instance, $AF_NN_Pe(t_1)$ is true if and only if (1) event e occurs on interval t_1, (2) the relation between the interval on which sentence $AN_Pe(t_1)$, conveying that information, is uttered and interval t_1, as the occurrence time of e, is an overlapping relation (on the right side of t_1), and (3) the relation between the interval on which

$AN_{Pe}(t_1)$ is uttered and the interval on which $AF_NN_{Pe}(t_1)$, conveying this last information, is uttered is also an overlapping relation (on the left side of the utterance time of the former sentence). Now, detensers take the equivalence between $AF_NN_{Pe}(t_1)$ and the conjunction of (1), (2), and (3) to be a reductionist reinterpretation of $AF_NN_{Pe}(t_1)$. Notice that (1), (2), and (3) are all *tenselessly true or tenselessly false!*

Now, an obvious objection is that there must be something wrong with the very idea of identifying the meaning of $AF_NN_{Pe}(t_1)$ with the meaning of the conjunction consisting of (1), (2), and (3), since $AF_NN_{Pe}(t_1)$, if true, is true only on some—not all—time intervals, whereas the conjunction of (1), (2), and (3), if true, is true tenselessly, that is, on all time intervals. But then, even worse, how can the equivalence between $AF_NN_{Pe}(t_1)$ and (1), and (2), and (3) be true, given that its left side cannot be true tenselessly, while its right side, if true, is true tenselessly? The answer to the last question is, of course, that $AF_NN_{Pe}(t_1)$, on the one hand, and (1), (2), and (3), on the other, relate as object language to metalanguage. In such a case, the question concerning conditions under which the equivalence holds is simply wrongly put. The equivalence does not mean that its left side and its right side must be true on the same intervals. It means, instead, that its left side is true on certain intervals (and only on them) if and only if its right side is true at all (and if at all, it is true on all the intervals).

But, no matter whether the question concerning the equivalence between $AF_NN_{Pe}(t_1)$ and the conjunction of (1), (2), and (3) can easily be answered, the objection concerning the meaning identification is serious. It must be admitted that the conjunction of (1), (2), and (3) is not a translation of $AF_NN_{Pe}(t_1)$. Then what is the equivalence for?

It is often said that, regardless of the fact that tensed sentences are not translatable into tenseless ones, the very fact that tensed sentences have tenseless truth conditions suffices to claim that tenses are not real. As already suggested, I think that the question is more complex. The (material) equivalence between $AF_NN_{Pe}(t_1)$ and the conjunction of (1), (2), and (3) only qualifies the right side of the equivalence to be a possible explanans of the fact that $AF_NN_{Pe}(t_1)$, if true, is a tensed truth. However, not only is there another possible explanation of the same fact—the one based on the two-dimensional representation and the flow of time—but it is only the comparison between the two possible explanations that clarifies the sense in which tenses are to be said to be real according to one of them and unreal according to the other. Namely, neither of the two explanations can reasonably deny the fact that there are tensed truths, and both can leave the tense logic system standing as it is. The difference between the two explanations lies only in the fact that one of them presupposes the flow of time and the other does not, so that tenses may be said to be unreal according to the latter in the sense in which they are real according to the former.

Let us remember an analogous example mentioned in the introduction. When I say that something (or somebody) is beautiful, I mean that that something (or somebody) has the property of *being beautiful*. Advanced psychology (or perhaps neurophysiology) can reinterpret my statement in terms of my reaction (or my brain's reaction) to that something (or somebody). Now, even though the original meaning is not the same as the meaning of the psychological (or neurophysiological) reinterpretation, psychologists (or neurophysiologists) may still deny that what I assert to be beautiful actually has such a property, that is, the property of being beautiful per se. But they would certainly not say that I am *wrong* when I say of something (or somebody) that it (he or she) is beautiful. They may only make a contrast between two interpretations and claim that the property of *being beautiful* is, according to their interpretation, not real in the sense in which it is real according to the naive understanding.

Ceteris paribus, Occam's razor favors the detensers' reinterpretation, but this reinterpretation can still seem unnatural because of our familiarity with tenses. The following analogy may help us get rid of such a feeling. It seems obvious that by saying of something that it is *here*, I am not ascribing to it a monadic property of *being here*. Instead, I am only saying that that something is there where I am. However, in the Hopi language, which is different from the standard average European language in a way that is important for our purposes, different "tenses" are used for saying that something is occurring *here* and for saying that something is occurring at some distant place (see Whorf 1956, p. 53). A speaker of such a language could be prone to believe that there is, in reality, a difference between *here* and *there* that corresponds to the difference between *presentness* and *futurity* according to our naive tensers' understanding of the difference between tenses. Curiously enough, such a

speaker could easily be charged by naive tensers in view of time to be a naive tenser in view of space!

Generally speaking, the so-called naturalistic fallacy consists in one's wrong belief that something is a property of reality in itself (see Frankena 1939). After their detensing analysis, detensers may charge tensers with committing such a fallacy.

10.5 Time and Modalities

Giving credit to the detensers' reinterpretation, I shall now try to differentiate modalities by introducing modal operators into a temporal system that contains neither tense operators nor tense predicates. This means that the system contains initially only tenseless sentences like $Ae(t_1)$. The axioms will be those cited in appendix A.

To differentiate modalities means, in this context, to express formally the difference between *deterministic* and *indeterministic* events.

Contrary to what we had to do in sketching the system of events in section 10.3, where the presence of both dates and tenses required that tense letters be placed *after* the sentence-forming operator *A*, here we can treat the symbols $\Box$ and $\Diamond$ as standard modal operators and place them *in front of A*. Namely, the fact that an otherwise well-specified event *e* occurs on t_1 according to a deterministic pattern (of any kind whatsoever) can be expressed by saying that the sentence $Ae(t_1)$, though logically contingent, is still necessarily true, which is formally to be written as $\Box Ae(t_1)$. Similarly, the fact that the occurrence of *e* on t_1 is precluded is to be put as $\Box\neg Ae(t_1)$. Generally, if a universe is deterministic in view of whatever happens on t_1, it should hold that $\Box AE(t_1) \vee \Box\neg AE(t_1)$, where *E* is a schematic letter substitutable by $e, e', e'', \ldots$. Generalizing the last condition so as to cover all intervals, that is, so as to become the axiom holding in a completely deterministic universe, we obtain $(t_n)(\Box AE(t_n) \vee \Box\neg AE(t_n))$.[7]

Now, in order to get the intended interpretation, we can use the possible-worlds semantics and take the set of possible worlds to be the set of all the worlds that are noncontradictorily describable by atomic sentences claiming various couplings between events and time intervals. Then, if the world for which $\Box Ae(t_1) \vee \Box\neg Ae(t_1)$ holds is a world accessible from some other world *w*, it is the only world on t_1 that is

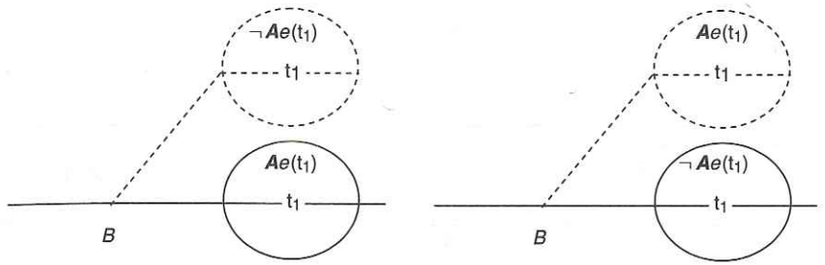


Figure 10.2

accessible from *w*. This means that if we take it that a real world consists of its time segments, the accessibility relation is an equivalence relation definable on the set of those possible worlds that are time segments of that real world (for more about this, see Arsenijević 2002, sec. 4). Consequently, within any completely deterministic real world the allegedly different modalities de facto collapse into just one of them—reality.[8]

In a noncompletely deterministic universe, there must be a sentence—at least one—such that, for some value of t_n, $\Box AE(t_n) \vee \Box\neg AE(t_n)$ does not hold. Let $Ae(t_1)$ be such a sentence. In view of this sentence, $\Diamond Ae(t_1) \wedge \Diamond\neg Ae(t_1)$ should hold. Let us consider how $\Diamond Ae(t_1) \wedge \Diamond\neg Ae(t_1)$ is to be interpreted according to the standard possible-worlds semantics.

$\Diamond Ae(t_1) \wedge \Diamond\neg Ae(t_1)$ means that there are at least two different accessible possible worlds, one in which $Ae(t_1)$ is true and $\neg Ae(t_1)$ false, and the other in which $\neg Ae(t_1)$ is true and $Ae(t_1)$ false. To meet such a condition, both accessible possible worlds must be of the same modal status. This is not the case if one of the two worlds is supposed to be *actual* and the other *merely possible* (see figure 10.2, where, after the branching point, *B*, the solid line depicts the actual world, and the dotted line the relation to a merely possible world). So, the required condition would be met either in the case in which both possible worlds were supposed to be actual or in the case in which both were supposed to be *merely possible* and *not yet actual* (see figure 10.3). The first option requires Lewis's plurality-of-real-worlds assumption (see Lewis 1986). However, I do not want to endorse such an assumption, not only for ontological but also for methodological reasons. Namely, I want to offer

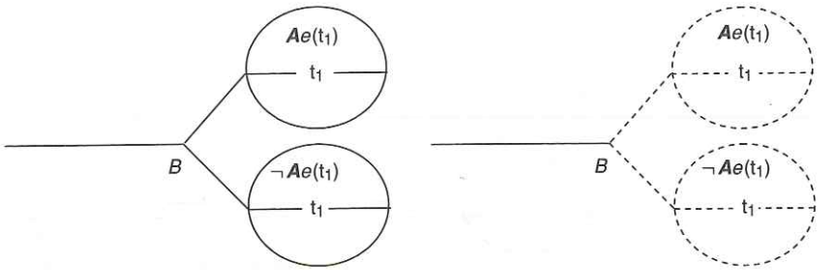


Figure 10.3

a *weaker* explanation for the case that there is just one real world, without precluding the possibility that there are more real worlds. How to fulfill the second option requirement?

The resolution is at hand. Since we are dealing with a *temporal* modal system, the two sentences $\Diamond Ae(t_1)$ and $\Diamond\neg Ae(t_1)$ are reasonably to be considered true on all the intervals that precede t_1. For the assumption that event $e(t_1)$ is indeterministic may be understood as implying that there is nothing in reality that would make either $\Diamond Ae(t_1)$ or $\Diamond\neg Ae(t_1)$ true *simpliciter* on any interval that precedes t_1. In addition, both $\Diamond Ae(t_1)$ and $\Diamond\neg Ae(t_1)$ remain true on all the intervals included in t_1 but not ending together with t_1, given that nothing has happened on them that makes $Ae(t_1)$ false. But on all the intervals that end together with t_1 or are later than t_1, either $Ae(t_1)$ is true *simpliciter* and $\neg Ae(t_1)$ no longer accessible, or $\neg Ae(t_1)$ is true *simpliciter* and $Ae(t_1)$ no longer accessible, so that on all those intervals either $\Diamond Ae(t_1)$ or $\Diamond\neg Ae(t_1)$ is true, and never both. In view of all these facts, the fact that a universe is completely indeterministic should be expressed through

$(t_n)((\Diamond AE(t_m) \wedge \Diamond\neg AE(t_m) \Leftrightarrow$
$\Leftrightarrow t_n \prec t_m \vee (\exists t_k)(t_n \{ t_k \wedge t_k \subset t_m \wedge (AE(t_k) \Rightarrow AE(t_m)))).$[9]

Note that, by interpreting the sentences about indeterministic events, I did not reject the principle of bivalence (as Łukasiewicz (1920) did). $Ae(t_1)$ and $\neg Ae(t_1)$ are always considered true or false, but on certain intervals *in a qualified sense* (in relation to accessible possible worlds) and on certain other intervals *simpliciter*. (Compare the diagrams in figure 10.4, where $Ae(t_1)$ and $\neg Ae(t_1)$ are, in the first, both true and false

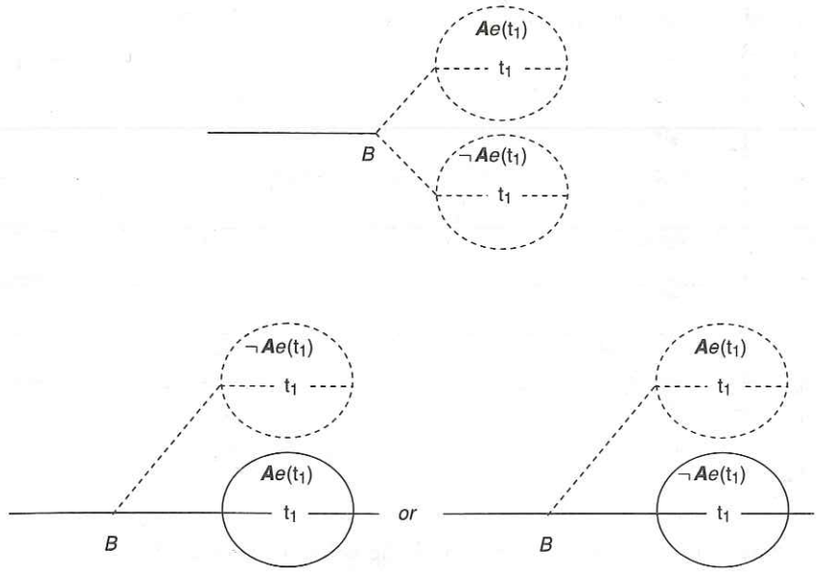


Figure 10.4

—but in different possible worlds—and where, in the second and in the third, $Ae(t_1)$ is true *simpliciter* and $\neg Ae(t_1)$ inaccessible *per accidens*, and $\neg Ae(t_1)$ is true *simpliciter* and $Ae(t_1)$ inaccessible *per accidens*, respectively.)[10]

Now, in a highly interesting contrast to the fact that the tense logic system sketched above could have been reinterpreted without the flow-of-time assumption, the tenselessly formalized system concerning indeterministic events cannot be consistently interpreted without such an assumption! For there is no other way to reconcile the fact that on some intervals $Ae(t_1)$ and $\neg Ae(t_1)$ are both possible (each true in view of an accessible possible world) with the fact that on some other intervals one of the two is true *simpliciter* and the other false *simpliciter* (i.e., true in an inaccessible possible world only). Namely, it is only the flow of time that can change the modal status of the possible worlds, in which $Ae(t_1)$ and $\neg Ae(t_1)$ are true in a qualified sense, in such a way that one of them ceases to be accessible. Notice that in order to represent such a change we need two different diagrams: one in which the actual world is connected by dotted lines with the possible worlds in which $Ae(t_1)$ and

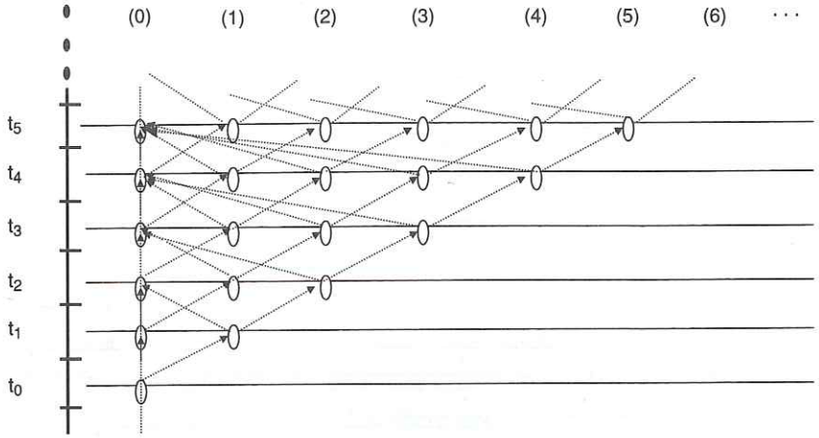


Figure 10.5

$\neg Ae(t_1)$ are true, respectively, and the other in which either $Ae(t_1)$ or $\neg Ae(t_1)$ belongs to the actual world, while the other, which doesn't, can no longer be connected with the actual world at all (see figure 10.4). In other words, the one-dimensional representation does not suffice.

If the existence of indeterministic events is not precluded, we can imagine a device—let us call it the *ontological wheel of fortune*—that would enable God to know which time is the present time by simply being informed about the number of possibilities at any time at which this number is greater than 1 (see figure 10.5). Let $t_0, t_1, t_2, \ldots$ be time intervals such that each of them abuts the one whose subscript is by one less than its own subscript, t_0 being the starting one. The wheel starts working at t_0 and functions by showing the numeral 0 at t_0 and then the numerals $0, 1, 2, 3, \ldots$ at any time so that if it shows k at t_n, it can show only 0 or $k+1$ at t_{n+1}, where the outcome is not predetermined in any other way. It is clear that at t_0 the number of possible outcomes at t_n is $n+1$, whereas at t_1, the number of possible outcomes at t_n is n (if $n > 0$), at t_2 it is $n-1$ (if $n > 1$), and so on, decreasing and sooner or later becoming equal to 1.

The branching possibilities are represented in terms of a B-series. But let us raise the following, only B-series-dependent question: what is the number of possibilities at, say, t_7? There is no answer that can be given independently of referring to t_0, or t_1, or t_2, or..., or t_7. The number of possibilities is 8 at t_0, 7 at t_1, 6 at $t_2, \ldots, 2$ at t_6, 1 at t_7 (as well as for any subscript greater than 7). This means that if the number of possibilities *really changes* i.e., really decreases from 8 to 1), then there is a *real flow of time* from t_0 to t_7. What holds for t_7 holds mutatis mutandis for any subscript n ($n > 0$). This means, in effect, that the endless functioning of our *ontological wheel* starting at t_0 implies the endless flow of time starting at t_0.

The passage of time has nothing to do with a metrics of time: t_0 can be a day, t_1 a second, t_3 a minute, and so on ad libitum. So, sitting outside the world, as it were, God could infer, using an appropriate ontological wheel, that t_n is the present day (minute, second, or whatever), by being informed that the number of possible outcomes at t_{n+k}, for $k > 0$, is $k+1$, just as we can infer that the number of possible outcomes at t_{n+k} is $k+1$, by perceiving the event $e(t_n)$ as an event occurring now (be this *now* a day, a minute, a second, or whatever).

Moreover, sitting outside a set of worlds, God could infer, using more ontological wheels installed in each of them, which time (day, minute, second, etc.) is the present time in each of them.[11] In particular, He could say for two otherwise indistinguishable worlds that they differ just in view of which time is the present time in each of them.[12] Such a distinction would mean that tenses are real. In effect, such a distinction would mean that by referring to an event as a *present* event, we do refer, inter alia, to an *objective monadic property* of it.

Accepting that the number of outcome possibilities related to an ontological wheel really changes in view of some given time, we must also accept that there is a real flow of time, that is, that the detensers' Occam's razor is no longer applicable because the *praeter necessitatem* clause no longer holds. The temporal-modal logic incorporating indeterministic events cannot tell us which time is present, but its interpretation requires the reality of tenses.

10.6 Conclusion

Not only is "Mellor's world" void of *tenses*, it is also void of *real possibilities*.[13] Detensers cannot speak of real possibilities, that is, of in-the-

world-inherent possibilities,[14] for if $Ae(t_1)$ is tenselessly true, it is true *simpliciter* on all the intervals, so that $\neg Ae(t_1)$ never belongs to an accessible possible world. In other words, all events are viewed *as if* they had already happened. However, it begs the question to argue that on all intervals either $Ae(t_1)$ is true *simpliciter* or $\neg Ae(t_1)$ is true *simpliciter* and give no other reason for this except that *on* t_1 either $Ae(t_1)$ is true *simpliciter* or $\neg Ae(t_1)$ is true *simpliciter*.

Given that $Ae(t_1)$ is tenselessly true, detensers can say that $\neg Ae(t_1)$ *could have been* true, for $Ae(t_1)$ is not a truth of logic. But they cannot distinguish between 'it can be otherwise' and 'it could have been otherwise',[15] for it is only the flow of time that changes an accessible possible world into a possible but inaccessible world. That's why detensers can speak of "indeterminism" only in a deviant sense that is not in accordance with the "core meaning" of the term.[16]

The tenseless theory of time is a consistent theory. But it precludes too much, or at least too much for those who as physicists believe in really indeterministic events, who as libertarians believe that our actions are really free, who as theologians believe in God's real intervention in history, or who simply as philosophers believe that all those things in which physicists, libertarians, and theologians believe are, even if not real, then at least possible.

Neither the tensed theory nor the tenseless theory can be simply refuted. But it is good to be aware of their assets and liabilities. Once the *praeter necessitatem* clause in Occam's *methodologico-ontological principle* has ceased to be fulfilled, tensers seem to gain the upper hand.

Appendix A

The axioms of the standard but interval-based time topology

A_1: $(t_n)\neg(t_n \prec t_n)$

A_2: $(t_k)(t_l)(t_m)(t_n)(t_k \prec t_m \wedge t_l \prec t_n \Rightarrow t_k \prec t_n \vee t_l \prec t_m)$

A_3: $(t_m)(t_n)(t_m \prec t_n \Rightarrow t_m \{ t_n \vee (\exists t_l)(t_m \{ t_l \wedge t_l \{ t_n))$

A_4: $(t_k)(t_l)(t_m)(t_n)(t_k \{ t_m \wedge t_k \{ t_n \wedge t_l \{ t_m \Rightarrow t_l \{ t_n)$

A_5: $(t_k)(t_l)(t_m)(t_n)(t_k \{ t_l \wedge t_l \{ t_n \wedge t_k \{ t_m \wedge t_m \{ t_n \Rightarrow t_l = t_m)$

A_6: $(t_m)(\exists t_n)t_m \prec t_n$

A_7: $(t_m)(\exists t_n)t_n \prec t_m$

A_8: $(t_m)(\exists t_n)t_n \subset t_m$

A_9': For any infinite sequence of intervals $t_1 t_2 \ldots t_i \ldots$ ordered by $\prec$, it holds that $(\exists u)(\bigwedge_{i<\omega} t_i \prec u) \Rightarrow (\exists v)(\bigwedge_{i<\omega} t_i \prec v \wedge (w)(\bigwedge_{i<\omega} t_i \prec w \Rightarrow \neg(\exists x)(x \prec w \wedge \neg x \prec v)))$

A_9'': For any infinite sequence of intervals $t_1 t_2 \ldots t_i \ldots$ ordered by $\succ$, it holds that $(\exists u)(\bigwedge_{i<\omega} t_i \succ u) \Rightarrow (\exists v)(\bigwedge_{i<\omega} t_i \succ v \wedge (w)(\bigwedge_{i<\omega} t_i \succ w \Rightarrow \neg(\exists x)(x \succ w \wedge \neg x \succ v)))$

(The last two axioms are formulated in the language $L_{\omega_1\omega}$.)

Prior's axiom holding for the so-called elementary events

A_{10}: $(t_n)(Ae(t_n) \Rightarrow (t_m)(t_m \subset t_n \Rightarrow Ae(t_m))$

Appendix B

The recursive definition of the truth conditions for the atomic sentences containing iterated tense predicates

$AT_kT_{k-1}\ldots T_2T_1e(t_n)$, for $k > 1$—where $T_1, T_2, \ldots, T_{k-1}, T_k$ stand for tense operators and $t_1, t_2, \ldots, t_{k-1}, t_k$ are variables implicitly present in the application of $T_1, T_2, \ldots, T_{k-1}, T_k$, respectively—is

1. false for any value of t_k (i.e., on all the intervals) *if* $AT_{k-1}\ldots T_2T_1e(t_n)$ is false (i.e., $AT_kT_{k-1}\ldots T_2T_1e(t_n)$ is false for any valuation for which $AT_{k-1}\ldots T_2T_1e(t_n)$ is false);
2. true, for any given valuation for which $AT_{k-1}\ldots T_2T_1e(t_n)$ is true,
   a. for all values of t_k such that t_k precedes t_{k-1}—if T_k stands for *F*;
   b. for all values of t_k such that t_k overlaps with t_{k-1}—if T_k stands for F_N;
   c. for all values of t_k such that t_{k-1} is included in t_k but where no subinterval of t_k is later than t_{k-1}—if T_k stands for *F-N*;
   d. for all values of t_k such that t_k is either identical with or included in t_{k-1}—if T_k stands for *N*;
   e. for all values of t_k such that t_{k-1} is included in t_k but where there are both subintervals of t_k that are later than t_{k-1} and those that are earlier than t_{k-1}—if T_k stands for *F-N-P*;
   f. for all values of t_k such that t_{k-1} is included in t_k but where no subinterval of t_k is earlier than t_{k-1}—if T_k stands for *N-P*;

g. for all values of t_k such that t_{k-1} overlaps with t_k—if T_k stands for N_P;

h. for all values of t_k such that t_{k-1} precedes t_k—if T_k stands for P.

Four lemmas provable by the use of mathematical induction

Lemma 1: Any closure of the open sentence $AT_kT_{k-1}\dots T_2T_1e(t_n)$ through at least one universal quantifier related to one of the variables $t_1, t_2, \dots, t_{k-1}, t_k$ gives a sentence that is false.

Proof. According to the truth conditions for the elementary tenses, $AT_1e(t_n)$ is *not* true for all values of t_1. But then, according to the truth conditions for the iterated tenses, $AT_2T_1e(t_n), \dots, AT_{k-1}\dots T_2T_1e(t_n)$, $AT_kT_{k-1}\dots T_2T_1e(t_n)$ cannot be true for all values of t_1 either. This represents the inductive basis. For the inductive step, let us suppose that $AT_i\dots e(t_n)$, where $1 \leq i < k$, is true for some valuation. Then, according to the truth conditions for iterated tenses, $AT_{i+1}T_i\dots e(t_n)$ is true for some but not for all values of t_{i+1}, and so, for values for which $AT_{i+1}T_i\dots e(t_n)$ is false, $AT_kT_{k-1}\dots T_2T_1e(t_n)$ is false also. This means that any occurrence of the universal quantifier related to one of the variables $t_1, t_2, \dots, t_{k-1}, t_k$ gives a sentence that is false.

Lemma 2: If $(\exists t_n)Ae(t_n)$ is true, then $(\exists t_k)(\exists t_{k-1})\dots(\exists t_2)(\exists t_1)(\exists t_n)AT_kT_{k-1}\dots T_2T_1e(t_n)$ is true, for whatever tense predicates $T_1, T_2, \dots, T_{k-1}, T_k$ stand for, and for any k.

Proof omitted

Lemma 3: If the sequences of tense predicates $T_kT_{k-1}\dots T_2T_1$ and $T'_kT'_{k-1}\dots T'_2T'_1$ differ at one place at least, there is no valuation for which both $AT_kT_{k-1}\dots T_2T_1e(t_n)$ and $AT'_kT'_{k-1}\dots T'_2T'_1e(t_n)$ are true.

Proof. Inductive basis: For any valuation for which $Ae(t_n)$ is false, both $AT_1e(t_n)$ and $AT'_1e(t_n)$ are false, and so are $AT_kT_{k-1}\dots T_2T_1e(t_n)$ and $AT'_kT'_{k-1}\dots T'_2T'_1e(t_n)$; while for any valuation for which $Ae(t_n)$ is true, but T_1 and T'_1 do not stand for the same tense predicate, $AT_1e(t_n)$ and $AT'_1e(t_n)$ cannot both be true, so that either both $AT_kT_{k-1}\dots T_2T_1e(t_n)$ and $AT'_kT'_{k-1}\dots T'_2T'_1e(t_n)$ are false or, if $AT_kT_{k-1}\dots T_2T_1e(t_n)$ is true, $AT'_kT'_{k-1}\dots T'_2T'_1e(t_n)$ is false, and vice versa. Inductive step: For any valuation for which both $AT_i\dots e(t_n)$ and $AT'_i\dots e(t_n)$ are true $(1 \leq i < k)$, but T_{i+1} and T'_{i+1} do not stand for the same tense predicate, $AT_{i+1}T_i\dots e(t_n)$ and $AT'_{i+1}T'_i\dots e(t_n)$ cannot both be true, so that either both $AT_kT_{k-1}\dots T_2T_1e(t_n)$ and $AT'_kT'_{k-1}\dots T'_2T'_1e(t_n)$ are false or, if $AT_kT_{k-1}\dots T_2T_1e(t_n)$ is true, $AT'_kT'_{k-1}\dots T'_2T'_1e(t_n)$ is false, and vice versa.

Lemma 4: There is no common value for t_{k-1} and t_k $(k > 1)$ that could render both $AT_kT_{k-1}\dots T_2T_1e(t_n)$ and $AT_{k-1}\dots T_2T_1e(t_n)$ true, except when T_k stands for N.

Proof. Let us suppose that $AT_{k-1}\dots T_2T_1e(t_n)$ is true for some valuation in which t_{k-1} is substituted for t_{k-1}. It immediately follows from the truth conditions concerning iterated tense predicates that any further application of a tense predicate that is not N moves us left or right along the time axis so that $AT_kT_{k-1}\dots T_2T_1e(t_n)$ must be false under the same valuation when t_{k-1} is substituted for t_k.

Notes

1. See Mellor 1981, chap. 4. Mellor's book was followed by a great number of articles, monographs, and collections directly or closely related to the debate between the tensed and tenseless theories of time (see Faye 1989; Farmer 1990; Le Poidevin 1991; Smith 1993; Oaklander and Smith 1994).

2. For various assessments of McTaggart's proof, see Oaklander and Smith 1994, pt. II, and Ludlow 1999, secs. 7.4, 8.5.

3. For comments and explanations of the meaning of these axioms, see the appendix of Arsenijević 2002.

4. My reason for using the phrase 'on an interval' in the case of an event occurring on (rather than in) an interval is that it has a connotation analogous to that in mathematics—namely, I want to say that the event occupies the whole interval (and not just a part of it).

5. If we represent the flow of time producing an interval as the motion of a point, we can reconcile this representation with the fact that, in our system, there is no such thing as *absolute presentness* by reinterpreting appropriately Whitehead's famous formula, "There is no continuity of becoming, though there is the

becoming of continuity" (for its original meaning, see Whitehead 1971, chaps. 3–4). Namely, however the flow of time is imagined to "produce" time's basic stuffs, they themselves are never extensionless.

6. This is the common feature of various detensing analyses: via token-reflexive truth conditions of tensed sentences (see Mellor 1998, sec. 3.2), via their utterance dates (see Smart 1980), via a co-reporting nature of tensed and tenseless sentences (see Beer 1994, pp. 91–93), via a contextualization of tensed sentence types (see Paul 1997, pp. 62ff.).

7. Since the last formula would be true not only when interpreted in an existing deterministic world but also in the case that no real world existed or nothing happened in the existing world, we can choose, in order to satisfy the Leibnizian requirement, an event—say, e—that supposedly really occurs on some interval—say, t_2—in the world assumed to exist, and add $Ae(t_2)$ as an antecedent of the above conjunction, so that the deterministic axiom becomes

Given that $Ae(t_2)$ is true, $Ae(t_2) \Rightarrow (t_n)(\Box AE(t_n) \wedge \Box AE(t_n))$.

Though the Leibnizian requirement is not yet fulfilled in the strong sense, since the truth of the second disjunct does not imply that something happens on t_n, the so-called modal version of the requirement *is* met (see Newton-Smith 1980, p. 47). Via $e(t_2)$, we are connected with a particular real world, and all other time intervals, regardless of whether they are "full" or "empty," are at least topologically determined via t_2.

8. Lewis (1973, p. 8) calls the "necessity in respect of all facts ... fatalistic necessity." However, at the most general level, at which we take into account just the core meaning of the term, other kinds of determinism are not distinguishable from fatalism.

9. In order to fulfill the Leibnizian requirement, we may again suppose that, for some e and some t_5, $Ae(t_5)$ is true *simpliciter* (which only means that we are speaking from within a world in which $e(t_5)$ is supposed to be actualized, and not that $e(t_5)$ is actualized according to a deterministic pattern), and we may formulate the indeterministic axiom as follows:

Given that $Ae(t_5)$ is true *simpliciter*, $Ae(t_5) \Rightarrow ((t_n)(\Diamond AE(t_m) \wedge \Diamond \neg AE(t_m) \Leftrightarrow t_n \{ t_m \vee (\exists t_k)(t_n \{ t_k \wedge t_k \subset t_m \wedge (AE(t_k) \Rightarrow AE(t_m)))))$.

10. The medieval logicians used the expression 'necessity *per accidens*' to denote something that is not logically necessary, but yet necessary in the sense that some real fact precludes it. However, in order to avoid ambiguity, I do not say that $Ae(t_1)$ (or $\neg Ae(t_1)$) is *impossible per accidens*, but only that the world in which it is true is *inaccessible per accidens*. An additional advantage is that by saying that $Ae(t_1)$ (or $\neg Ae(t_1)$) is inaccessible *per accidens*, we do not preclude the possibility that the world in which $Ae(t_1)$ (or $\neg Ae(t_1)$) holds is a *real* world. We say only that this world is no longer accessible *from within the world* in which it happened that something de facto precludes the truth of $Ae(t_1)$ (or $\neg Ae(t_1)$).

11. By contrast, consider Oaklander's (1994, p. 326) example, where God, assumed to be outside time, is looking at all facts in the world without being able to take into account in-the-world-inherent modalities.

12. See Mellor 1998, pp. 19ff., where, owing to the absence of in-the-world-inherent modalities, the same example is used to show that there is no real difference between the two worlds.

13. For the concept of *real possibility*, see Deutsch 1990.

14. For the conception of a *real world full of different modalities*, see Stalnaker 1976.

15. Le Poidevin (1991, p. 130) admits that "the future cannot be *ontologically indeterminate*," but only "*epistemologically indeterminate*." By contrast, see Rescher 1968.

16. See Mellor 1998, secs. 10.2 and 11, where determinism is understood as presupposing the existence of a causal chain between events. But then, the difference between deterministic and indeterministic events is still not a difference in view of the fact that, on the tenseless view, particular events and particular time intervals are either "coupled" once and for all or "noncoupled" once and for all.

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Section B

Tensed Theories of Time